# SELF SIMILARITY SOLUTIONS FOR SPHERICAL AND CYLINDRICAL SHOCK WAVES IN MAGNETO DYNAMICS

**THESIS**

**SUBMITTED TO THE**

**BUNDELKHAND UNIVERSITY, JHANSI**
**FOR THE AWARD OF THE DEGREE OF**
**DOCTOR OF PHILOSOPHY**
**IN**
**MATHEMATICS**



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# CERTIFICATE

This is to certify that thesis entitled ***"Self similarity solutions for spherical and cylindrical shock waves in magneto dynamics"*** which is being submitted by Mr. Jitendra Kumar is worthy of consideration for the award of degree of Doctor of Philosophy in mathematics and is a record of the original and bonafied research work carried out by him under my guidance and supervisions.

The thesis has reached the standard fulfilling the requirement of regulating relating to the degree. The result presented in this thesis have not been submitted in part or fully to any other university or institution for award of degree or diploma. Mr. Jitendra Kumar had been worked out for more than 210 days in the Department of mathematics Bipin Bihari P.G. College, Jhansi.

18/08/09

**(Dr. Kishore Kumar Srivastava)**

# DECLARATION

I hereby declare that the thesis entitled ***"Self similarity solutions for spherical and cylindrical shock waves in magneto dynamics"*** being submitted for the degree of Doctor of Philosophy to the Bundelkhand University (U.P.) is an innovative piece of work carried out with utmost dedication by me and to the best of my knowledge and belief it has not been submitted else where.

Place : Jhansi

Date : July 2009

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Jitendra soni
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# LIST OF NOTATION

| | |
|---|---|
| P | Pressure |
| $\rho$ | density |
| T | Temperature |
| $\bar{v}$ | Velocity vector |
| V | Volume |
| $\bar{r}$ | Position vector |
| t | Time |
| $\gamma$ | Specific heat ratio |
| Q | Heat added to unit mass |
| h | Enthalpy per unit mass |
| g | Gravitational acceleration |
| u,v,w | Cartesion components of velocity |
| x,y,z | Cartesion coordinates |
| $\delta_{ij}$ | Kronecker delta |
| $\in_{ijk}$ | Permutation symbol |
| $\bar{F}$ | Force vector |
| c | Velocity of light |
| a | Velocity of sound |
| E | Energy |
| $\Gamma$ | Gas constant |
| r | Radial distance (radius) |
| G | Gravitational constant |
| $\eta,\xi$ | Similarity variable |

| | |
|---|---|
| $C_p$ | Specific heat at constant pressure |
| $C_v$ | Specific heat at constant volume |
| $\overline{H}$ | Magnetic field intensity |
| $\overline{E}$ | Electric field intensity |
| $\overline{B}$ | Magnetic induction vector |
| $\overline{D}$ | Electric displacement vector |
| $\bar{J}$ | Current density vector |
| $\sigma$ | Conductivity of the medium |
| $\mu$ | Permeability of the medium |
| q | Charge density of the medium |
| $\in$ | Dielectric constant of the medium |
| u | Velocity |
| F | Radiation heat flux |
| $M_A$ | Mach number |
| Pr | Material pressure |
| $\mu\,\upsilon$ | Mean-free path of radiation |
| v | Shock velocity |
| m | Mass |
| h | Magnetic field |
| $M_h$ | Magnetic Mach number |
| $H_\theta$ | Azimuthal magnetic field |
| $H_z$ | Transverse magnetic field |
| e | Material energy |
| N | Number of molecules in Gas |
| K | Boltzmann's constant |
| n | Number of gram moles in the Gas |

# *Preface*

The Present thesis an out come of researches carried out by me in the field of "Self similarity solutions for spherical & cylindrical shock waves in magnetodynamics" under the supervision of Dr. Kishore K. Srivastava, Head of Mathematics Department, Bipin Bihari (P.G.) College, Jhansi, affiliated to Bundelkhand University, Jhansi is being submitted for the award of Ph. D. degree in Mathematics. The thesis has been divided into eight chapters, each chapter has been subdivided into a number of articles.

The first chapter is introductory. It gives in brief, an idea about shock waves, spherical and cylindrical shock wave. Similarity principle, equation of motion and jump condition behind the shocks in magnetogasdynamics, radiation phenomennon, concept of self gravitation, magnetohydrodynamics waves, MHD and flow stability, MHD and Sun, characteristic method and Witham's rule.

The second chapter is devoted to unify study of self similar model of exponential spherical, cylindrical & plane shock wave taking magnetic radiative heat flux into account where total energy of the wave is variable and atmosphere is uniform.

The third chapter deals with self similar study of power driven isothermal flow behind cylindrical shock. A qualitative behaviour of gaseous mass has been investigated with the help of equation of motion taking monochromatic radiation with gravitational effect in non uniform atmosphere. A comparative study is made between flow variables. Numerical integration of flow variables is carried out through software matlab.

The chapter four, an attempt is made for the study of propagation of plane shock wave in magnetogasdynamics under isothermal condition where temperature gradient is zero and radiation effect have been taken out. Numerical integration of differential equation is carried out on DEC 1090, computer at IIT Kanpur, by R.K. G's program.

The chapter fifth deal with "Self similar solution of cylindrical shock wave in magnetogasdynamic". Taking gravitational force under non uniform atmosphere with radiation heat flux into account. The radiation pressure and radiation energy have been

ignored we have considered the problem shock wave reduced by a point explosion in a gas.

The chapter six an attempt is made for study of the "Analytical solution of spherical shock wave in a rotating gas with axial component of magnetic field". Taking density as constant ahead the shock front. By using C.C.W. method analytical solution are obtained for shock velocity and shock strength for weak and strong magnetic field. For strong shock, also we have considered two cases i.e. when the magnetic field is strong and when weak i.e. non magnetic case.

In seventh chapter. We discussed with self similar model of radiative shock wave in magnetohydrodynamics with magnetic effect. The model of self similar shock waves produce on account of an instantaneous release of energy in a non-uniform equilibrium conditions. The disturbance are headed by a shock of variable strength.

In eighth chapter an attempt is made for similarity solution of cylindrical shock wave with radiation energy and material pressure in magnetohydrodynamics. A thorough analysis of self similar equation of dynamics taking material pressure & radiation flux into account have been studied. The radiation pressure and radiation energy have been also consider. The gas in the undisturbed field, is assume to be at rest. It is grey and opaque in non uniform atmosphere.

# Content

# Chapter - I

## INTRODUCTION

The study of 'Fluid Dynamics' is directed to the behaviour of a fluid in motion. The liquid in motion, and gas states are referred to generally as fluids. Some of the notable examples of application are : flow of water through pipes; motion of an aircrafts or a missiles in the atmosphere. The study also yields the methods and devices for the measurement of various parameters, e.g. the pressure and velocity in a fluid at rest or in motion.

The first notable works on fluids appeared in the seventeenth century. In 1687, Newton in his book 'Principal' deals with the influx of a fluid, the resistance of a fluid and the resistance of projected bodies and the wave motion. Daniel Bernoulli, in 1738 determined the relation between pressure and velocity and formalized this in his theorem. In 1743 his son John Bernoulli applied the momentum principle to infinitesimal elements. Euler in 1755, realized the significance of the Bernoullis' work and used it as the basis, setting down the fundamental equations of motion and equation of continuity for an ideal inviscid fluid. An alternate form of these equations was given by Lagrange, in 1781 and 1789. Equations including viscosity were derived by Navier in 1822 and Stokes in 1845.

In 1858 Helmholtz published a paper on vortex motion and in 1868 another on free stream line potential flow.

Near the end of the nineteenth century, fluid flow itself began to be extensively observed and investigated. The modern trend in fluid dynamics is to investigate the flow of electrically conducting fluid at very high temperature.

At a microscopic level, the three states of a given substance are different because of the difference in the intermolecular distance. In many cases at normal conditions the molecular distance of a fluid are less than the minutest of any physical dimensions of practical interest. As a result, we interested in the statistical average properties and the behaviour of large numbers of molecules, and not in that of individual molecules. That is, macroscopic and not microscopic, properties are of interest. In fluid dynamics as individual molecules are not being considered, the fluid can be regarded as a continuous

medium and so the physical quantities such as mass, momentum etc. of the fluid contained in a very small volume are regarded as being spread uniformly throughout that volume.

In dealing with the gases at very low pressure, as in the upper atmosphere, or at very high temperatures such as in a plasma, the continuum concept of fluid dynamics must be violated and the study that has to be based on the behaviour of individual molecules (i.e. on the macroscopic approach) [1].

In our studies we frequently refer to a 'small element of fluid' which is always supposed so large that it still contains a very large number of molecules, as fluid is a continuous medium. So when we take of an infinitesimal element of volume we always mean that which are physically infinite small. Such an element is called a fluid particle.

In continuum dynamics we assume that the macroscopic fluid properties, for example mean density, mean pressure, vary continuously with (a) the size of element of fluid considered (b) the position in the fluid and (c) the time. In (a), the variation becomes negligible as the element is physically very small. Thus, fluid properties density, pressure and velocity are expressed as continuous functions of position and time only. On this basis, it is possible to establish equations governing the motion of a fluid, which are independent of the nature of the particle structure. So gases and liquids may be treated together.

There are two distinct methods of specifying the flow field [2].

**LAGRANGIAN METHOD:** In this method, the flow variables (velocity, pressure and density) of a selected fluid element or particle are described. If $\bar{r}_0$ is the position of the center of mass of the fluid element at time $t_0$ then the basis flow quantity in the Lagrangian description is the velocity $\overline{\nabla}(r_0, t)$. This method is also referred is individual same face of change.

**EULERIAN METHOD:** In this method, the flow quantities are described at all points of space occupied by the fluid at all times i.e. flow quantities are defined as the function of position in space ($\bar{r}$) and time (t). The basic flow quantity is the vector velocity $\overline{\nabla}(r, t)$. This method corresponds to 'local time rate of change'.

Consider any scalor function f (x,y,z,t) associated with some property of the fluid (it could be density, velocity etc.). Suppose fluid particle has the position (x,y,z) or P $(\bar{r})$ at time at time t. Keeping this point fixed the change in f is during the interval of time $\delta t$ is,

$$f(x, y, z, t + \delta t) - f(x, y, z, t)$$

Hence the 'local time rate of change' is given by,

$$\frac{\partial f}{\partial t} = \lim_{\delta t \to 0} \frac{f(x, y, z, t + \delta t) - f(x, y, z, t)}{\delta t}$$

As the point P is fixed the local time differential operator $\frac{\partial f}{\partial t}$ is not carried along by the moving fluid.

Now let at time $t + \delta t$ the fluid particle, which was at the position (x,y,z) originally, is in the position $(x + u\,\delta t,\ y + v\delta t,\ z + w\,\delta t)$ where u, v, and w be the velocity components at the position at time t. The corresponding change of f is given by,

$$f(x + u\,\delta t,\ y + v\delta t,\ z + w\delta t,\ t + \delta t) - f(x, y, z, t)$$

or $\quad f(\bar{r} + \delta\bar{r},\ t + \delta t) - f(\bar{r},\ t)$

and rate of change is,

$$\frac{df}{dt} = \lim_{\delta t \to 0} \frac{f(\bar{r} + \delta\bar{r},\ t + \delta t) - f(\bar{r},\ t)}{\delta t}$$

This gives the 'individual time rate of change'. As the point P is moving it gives the rate of change which is carried along by the moving fluid.

Since,

$$f = f(x, y, z, t)$$

$$\therefore\ df = \frac{\partial f}{\partial t} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz + \frac{\partial f}{\partial t} dt$$

So,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{\partial y}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

If $\bar{v} = [u, v, w]$ be the velocity of the fluid particle at P and

$$\frac{dx}{dt} = u, \ \frac{dy}{dt} = v, \ \frac{dz}{dt} = w$$

then,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} u + \frac{\partial f}{\partial y} v + \frac{\partial f}{\partial z} w$$

$$= \frac{\partial f}{\partial t} + \left( i\frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} + k \frac{\partial f}{\partial z} \right) . \left( ui + vj + wk \right)$$

or,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \left( \bar{v} . \nabla \right) f$$

where $\bar{v} = ui + vj + wk$

The first term on the right hand side represents the local rate of change of f, and the second term the convective rate of change.

If $\bar{v}$ is introduced for f in the above equation then the total derivative of velocity with respect to time is,

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{\partial \bar{v}}{\partial t} + \left( \bar{v} \ \nabla \right) \bar{v}$$

In cartesion rectangular coordinates

$$\left( \bar{v} . \nabla \right) = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

It is this term in the Euler relation for acceleration which is responsible for the non-linearities in the equations of motion of fluid dynamics.

## FUNDAMENTAL EQUATIONS

In order to study the details of the fluid flow, we wish to find the density distribution, pressure distribution, velocity distribution, states of the fluid etc. at all point of space occupied by the fluid at all times. Hence a knowledge of three velocity components (u,v,w), the temperature T, the pressure p, the density $\rho$, etc. of the fluid which are functions of position in space $(\bar{r})$ i.e. (x,y,z) and the time t, is needed. We

obtain relations connecting these unknowns and which would explain the particular problem of the fluid motion. We call these relations as fundamental equations.

The flow of a compressible, non-heat conducting and inviscid perfect fluid is described by the four physical variables pressure p, density $\rho$, vector velocity $\bar{v}$, and temperature T. So there must be four fundamental equations to find these variables. These are [3],

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \bar{v}) = 0$$

or

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$ in one dimension equations of motion,

$$\frac{D\bar{v}}{Dt} = \frac{\partial \bar{v}}{\partial t} + (\bar{v}.\nabla) = -\frac{1}{\rho}\nabla P$$

When the external force is negligible.

Energy equation is

$$\frac{DQ}{Dt} = \frac{Dh}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$$

For the adiabatic flow

$$\rho\frac{Dh}{Dt} - \frac{Dp}{Dt} = 0$$

and equation of state for perfect gas,

$$P = \rho RT$$

when there is no intermolecular attractions.

Now we consider the behaviour of the fluid flow at high temperature, where some of the fluids become conducting and the interaction between motions of the conducting fluids and variations in electromagnetic fields may not be negligible. To describe this interaction phenomena of the flow field with the electromagnetic field we find the velocity vector $\bar{v}$, pressure p, density $\rho$, temperature T, and the magnetic field vector $\overline{H}$, which are the functions of the special coordinates (x,y,z) and the time t. To calculate

these unknowns we must find the relations which are the fundamental equations for a conducting compressible fluid. In these equations there are coupling terms between the electromagnetic and fluid dynamics phenomena. We shall derive and discuss these equations [4], [5], [6].

## EQUATION OF CONTINUITY

This equation simply expresses the law of conservation of mass. The quantity of fluid entering a certain volume in space must be balanced by that quantity leaving i.e. matter is neither created nor destroyed.

We now formulate this principle mathematically.

Let V be any arbitrary volume fixed in space, bounded by a surface S, and containing a fluid of density $\rho$ . The volume element $\delta V$ is small so that $\rho$ can be regarded as constant through it.

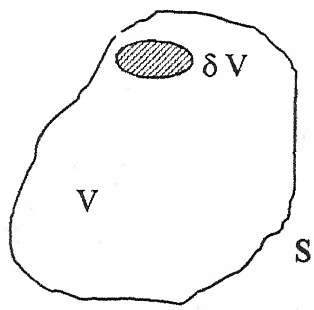


**Fig. 1.10**

The mass of the fluid with in the volume V is $\int_V \rho \, dV$. The rate of generation of the fluid within the volume is,

$$\frac{\partial}{\partial t} \int_v \rho \, dV = \int_v \frac{\partial \rho}{\partial t} dV$$

For $\rho \frac{\partial}{\partial t}(dV) = \rho \, d\left(\frac{\partial V}{\partial t}\right) = \rho_0 0 = 0$, as volume is constant with respect to time.

If the volume V occupied by a moving fluid, the fluid enters V through parts of its boundary surface S and leaves through another part.

Let $\hat{n}$ be a unit outward normal vector drawn on the surface element dS. The normal velocity is $\hat{n}\ \bar{v}$. The total outward flux is,

$$\int_S \rho\,(\hat{n}.\ \bar{v})\,dS = \int_s \hat{n}.\ (\rho\,\bar{v})\,dS$$

Using Gauss's theorem,

$$= \int_V \nabla\ .\ (\rho\,\bar{v})\,dV$$

The sum of the net outward convection of mass plus the rate of generation, of the fluid within the volume must be zero.

$$\text{or} \int_V \nabla\,(\rho\,\bar{v})\,dV + \int_V \frac{\partial\rho}{\partial t}\,dV = 0$$

$$\int_V \left[\frac{\partial\rho}{\partial t} + \nabla.\ (\rho\,\bar{v})\right] dV = 0$$

Since this is true for arbitrary elementary volumes

$$\frac{\partial\rho}{\partial t} + \nabla\ .\ (\rho\,\bar{v}) = 0 \qquad (1.01)$$

or,

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}\,(\rho\,v_i) = 0 \qquad i = 1,2,3$$

The Maxwell's electromagnetic equations for a conducting medium are,

$$\text{curl}\ \bar{E} = -\frac{\partial\bar{B}}{\partial t} \quad \text{where}\ \bar{B} = \mu\,\bar{H}$$

$$\text{curl}\ \bar{H} = 4\,\pi\,J + \frac{\partial\bar{D}}{\partial t}$$

$$\text{div}\ \bar{B} = 0$$

$$\text{and div}\ \bar{D} = 4\,\pi\,q$$

Differentiating the fourth Maxwell's equation

$$\text{div}\left(\frac{\partial\bar{D}}{\partial t} = 4\pi\,\frac{\partial q}{\partial t}\right)$$

Using the second Maxwell's equation it becomes,

$$\text{div}\ \left(\text{curl}\ \bar{H} - 4\,\pi\,\bar{J}\right) = 4\,\pi\,\frac{\partial q}{\partial t}$$

Third Maxwell's equation is

$$\text{div } \overline{H} = 0$$

We have,

$$-4\pi \text{ div } \bar{j} = 4\pi \frac{\partial q}{\partial t}$$

or $$\frac{\partial q}{\partial t} + \text{div } \bar{J} = 0$$

If the charged particle moves with velocity $\bar{v}$

$\bar{J} = q\bar{v}$ then,

$$\frac{\partial q}{\partial t} + \text{div } (q\,\bar{v}) = 0$$

This is the equation of continuity for the electric charge moving under the effect of magnetic field which is similar to the equation (1.1).

## EQUATION OF STATE

The electrodynamical state relation is a simple relation between the current density $\bar{J}$, fields and fluid motion. or the ohm's law is,

$$\bar{J} = \sigma\left\{\overline{E} + \mu\,\bar{v} \times \overline{H}\right\} + q\,\bar{v}$$

Where $\sigma$ is the electrical conductivity and $q\bar{v}$ is the current depending on the motion of the net charge q.

The state of a compressible fluid is defined by the pressure p, the entropy S, the internal energy $\varepsilon$, the absolute temperature T, the mass density $\rho$ and the specific volume V.

An equation of state is formed by expressing any one of the thermodynamic variables in terms of the other two quantities mentioned above.

For example,

$$P = P(\rho, T)$$

or $$E = E(P, \rho)$$

For a perfect fluid (no intermolecular intractions) having negligible viscosity the equation is,

$$p = R\,\rho\,T$$

where, R is the universal gas constant.

Assuming the process to be adiabatic and isentropic we have,

$$\varepsilon = C_V\,T$$

$$\varepsilon = \frac{p}{R\rho} C_V \quad \text{as } p = \rho RT$$

Also $\quad C_p - C_v = R$

or $\quad \dfrac{C_p}{C_v} - 1 = \dfrac{R}{C_v}$

or $\quad \gamma - 1 = \dfrac{R}{C_v}$

$\therefore \quad C_v = \dfrac{R}{\gamma - 1}$

So,

$$\varepsilon = \frac{P}{\rho(\gamma - 1)}$$

This is the calorie equation of the state of the medium. $\varepsilon$ is the internal energy per unit mass.

When a fluid element changes its state isentropically the state equation is,

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0$$

## EQUATION OF MOTION

This equation express the laws of conservation of momentum in the fluid. The obtain equations deriving this we proceed as following

We consider a fluid mass in motion at time t occupying a volume V and bounded by a surface S. Let $\delta V$ be a small element of volume. If the fluid has the density $\rho$, the mass of the element is $\rho\delta V$ and it moves with velocity $\bar{v}\,(\bar{r},t)$

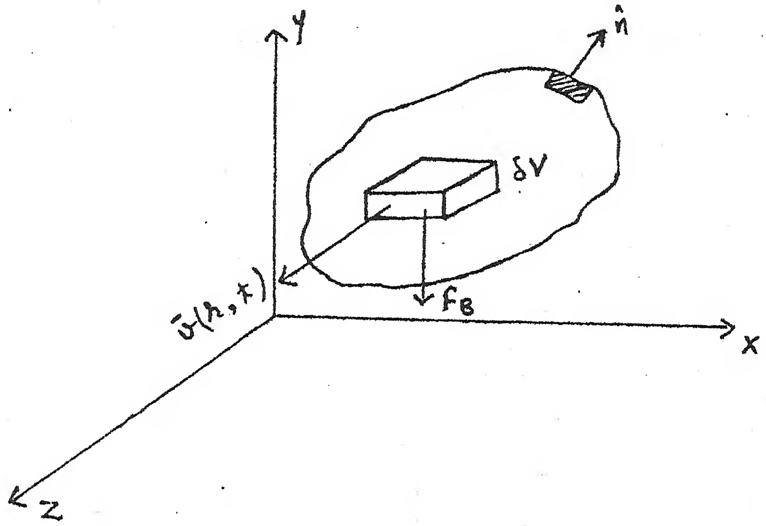


**Fig. 1. 02**

The inter force on the element is $\rho\delta V\left(\frac{D\bar{v}}{Dt}\right)$. This force equals the rate of change of linear momentum of the element. For the whole fluid mass,

$$F_I = \iiint \frac{D\bar{v}}{Dt}\,\rho\delta V$$

The force on the body is the sum of body and pressure force.

Consider a surface element $\delta S$ and that $\hat{n}$ be the unit outward normal vector to $\delta S$. Let p is the fluid pressure at any point on the element $\delta$ S. Then the force on it due to the fluid outside the particle is,

$P\,(-\,\hat{n})\,\delta S$ (For pressure acts along inward) Hence the total pressure force is,

$$-\iint_S \hat{n}\,p\ \ dS$$

using Gauss's theorem,

$$F_p = -\iiint_V \ \text{grad}\ p\ dV$$

When the fluid moves in an electric and magnetic field, the body force $F_B$ consists of three parts; gravitational, electric and magnetic.

The gravitational body force on the element is $\bar{g}\rho\delta V$ where $\bar{g}$ is the gravitational acceleration.

The electric body force on it due to an electric field of intensity $\bar{E}$ would be $\bar{E}\ q\ \delta V$.

The current flowing through the element is $\bar{i} = \bar{J}\ \delta A$. Where $\delta A$ be normal cross section of fluid element.

Hence using Biot-Savart law, magnetic force in the element is,

$$\bar{i} = \delta l \times \bar{B}$$

$$= \bar{J}\ \delta A\ \ \delta l \times \bar{B}$$

$$= (\bar{J} \times \bar{B})\delta V$$

$$= \mu\left(\bar{J} \times \bar{H}\right)\ \delta V$$

Hence the total body force $F_B$ on the fluid mass is,

$$F_B = \iiint_V \left(\rho\bar{g} + \mu\,\bar{J} \times \bar{H} + q\,\bar{E}\right) dV$$

Now,

$$F_I = F_B + F_p$$

So,

$$\iiint_V \left\{-\rho\frac{D\bar{v}}{Dt} + \left(\rho\,\bar{g} + \mu\,\bar{J} \times \bar{H} + q\,\bar{E}\right) - \mathrm{grad}P\right\} dV = 0$$

The above equation is true for arbitrary elementary volumes and so,

$$\rho\,\frac{D\bar{v}}{Dt} = -\ \mathrm{grad}P + \rho\,\bar{g} + \mu\,\bar{J} \times \bar{H} + q\,\bar{E}$$

This is the general equation of motion for a conducting fluid. Where Eulerian derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\bar{v}.\ \nabla)$$

The term involving $\overline{E}$ can be neglected as compare to the term involving magnetic field $\overline{H}$. As the order of magnitude of $q\overline{E}$ is very-very small as compare to the order of magnitude of $\mu\,\overline{J} \times \overline{H}$ i.e.

$$0\left(|q\overline{E}|\right) << 0\left(|\mu\,\overline{J} \times \overline{H}|\right)$$

So the modified form of equation of motion is,

$$\rho\,\frac{D\overline{v}}{Dt} = -\,\text{grad}\,p + \rho\,\overline{g} + \mu\,\overline{J} \times \overline{H}$$

We know, $\quad curl\ \overline{H} = 4\,\pi\,\overline{J} + \in \dfrac{\partial \overline{E}}{\partial t}$

The term $\in \dfrac{\partial \overline{E}}{\partial t}$ can be neglected as compare to curl $\overline{H}$ as,

$$0\left(\left|\in \frac{\partial \overline{E}}{\partial t}\right|\right) << \left(|\text{curl}\ \overline{H}|\right)$$

so,

$$\text{Curl}\ \overline{H} = 4\,\pi\,\overline{J}$$

Then the magnetic force is,

$$\mu\overline{J} \times \overline{H} = \frac{\mu}{4\pi}\ \text{curl}\ \overline{H} \times \overline{H}$$

or $$= -\frac{\mu}{4\pi}\,\overline{H} \times \text{Curl}\ \overline{H}$$

Since (using vector identity)

$$\overline{H} \times \text{curl}\ \overline{H} = \text{grad}\left(\frac{H^2}{2}\right) - \left(\overline{H}.\nabla\right)\overline{H}$$

Then,

$$\mu\,\overline{J} \times \overline{H} = -\,\text{grad}\left(\frac{\mu H^2}{8\pi}\right) + \frac{\mu}{4\pi}\left(\overline{H}.\nabla\right)\overline{H}$$

So equations of motion can be written as,

$$\rho\frac{D\overline{v}}{Dt} = -\ \text{grad}\ p + \rho\,\overline{g} - \text{grad}\left(\frac{\mu H^2}{8\pi}\right) + \frac{\mu}{4\pi}\left(\overline{H}.\nabla\right)\overline{H}$$

In summation convention form it can be written as,

$$\frac{\partial}{\partial t}(\rho v_j) + v_j \frac{\partial}{\partial x_i}(\rho v_i) + \left(v_i \frac{\partial}{\partial x_i}\right) v_j$$

$$= -\frac{\partial}{\partial x_i}\left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij} + \frac{\mu}{4\pi}\left(H_i \frac{\partial}{\partial x_i}\right) H_j$$

where

i is the dummy index

j is the free index

and i, j =1, 2, 3

## EQUATION FOR THE VARIATION OF THE MAGNETIC FIELD

This equation expresses how the magnetic field vary or the time dependence of the magnetic field.

The Maxwell's electromagnetic field equations for a conducting medium are,

$$\text{div}\,\overline{E} = 4\pi q$$

$$\text{div}\,\overline{H} = 0$$

$$\text{Curl}\,\overline{E} = -\frac{1}{c}\frac{\partial \overline{H}}{\partial t}$$

$$\text{and, Curl}\,\overline{H} = \frac{4\pi}{c}\overline{J} + \frac{1}{c}\frac{\partial \overline{E}}{\partial t}$$

The first two equations are derived by applying Gauss's theorem to a closed surface. Third equation is the mathematical formulation of the Faraday's law of induction and last equation describes how the magnetic field $\overline{H}$ depends on the conduction current $\overline{J}$ and the displacement current $\left(\frac{1}{c}\frac{\partial \overline{E}}{\partial t}\right)$.

$$\text{Since, Curl}\,\overline{H} = \frac{4\pi}{c}\overline{J} + \frac{1}{c}\frac{\partial \overline{E}}{\partial t}$$

By neglecting the term $\frac{1}{c}\frac{\partial \overline{E}}{\partial t}$ above equation reduces to,

$$\text{Curl}\,\overline{H} = \frac{4\pi}{c}\overline{J}$$

Taking Curl of this,

$$\text{Curl Curl } \overline{H} = \frac{4\pi}{c} \text{ Curl } \overline{J}$$

or $$\text{grad div } \overline{H} - \nabla^2 \overline{H} = \frac{4\pi}{c} \text{Curl} \left[ \sigma \left( \overline{E} + \frac{1}{c} \overline{v} \times \overline{H} \right) \right]$$

where, $\overline{J} = \sigma \left( \overline{E} + \frac{1}{c} \overline{v} \times \overline{H} \right)$

since, $\text{div } \overline{H} = 0$

So,

$$-\nabla^2 \overline{H} = \frac{4\pi\sigma}{c} \left[ \text{Curl } \overline{E} + \frac{1}{c} \text{ Curl } \left( \overline{v} \times \overline{H} \right) \right]$$

Also,

$$\text{Curl } \overline{E} = -\frac{1}{c} \frac{\partial \overline{H}}{\partial t}$$

$$-\nabla^2 \overline{H} = \frac{4\pi\sigma}{c^2} \left[ -\frac{\partial \overline{H}}{\partial t} + \text{Curl } \left( \overline{v} \times \overline{H} \right) \right]$$

or, $$\frac{\partial \overline{H}}{\partial t} = \text{Curl } \left( \overline{v} \times \overline{H} \right) + \frac{c^2}{4\pi\sigma} \nabla^2 \overline{H}$$

We may write,

$$\frac{\partial \overline{H}}{\partial t} = \text{Curl } \left( \overline{v} \times \overline{H} \right) + \eta \nabla^2 \overline{H}$$

where $\eta = \dfrac{c^2}{4\pi\sigma}$

or,

$$\frac{\partial \overline{H}}{\partial t} = (\overline{H}.\nabla)\, \overline{V} - (\overline{V}.\nabla)\, \overline{H} - \overline{H}\, (\nabla.\overline{V}) + \eta\, \nabla^2 \overline{H}$$

In summation convention form it can be written as.

$$\frac{\partial}{\partial t}(H_j) = (H_i \frac{\partial}{\partial x_i})\, v_i - (v_i \frac{\partial}{\partial x_i}) H_i - H_i\, (\frac{\partial}{\partial x_i} v_i) = \eta\, (\frac{\partial^2}{\partial x_{ii}^2})\, H_i$$

This is the magnetic field equation for a conducting compressible fluid.

For a fluid at rest it reduces to the diffusion equation.

$$\frac{\partial \overline{H}}{\partial t} = \eta \nabla^2 \overline{H}$$

$\eta$ is called magnetic diffusivity. For an equation of this type the rate of decay of the magnetic field is very rapid.

And when the conductivity is so large that the term $\eta \nabla^2 \overline{H}$ can be neglected that behaviour of the magnetic field is given by,

$$\frac{\partial \overline{H}}{\partial t} = \text{Curl} \left(\overline{v} \times \overline{H}\right)$$

or

$$\frac{\partial \overline{H}}{\partial t} = \left(\overline{H}.\nabla\right)\overline{V} - \left(\overline{v}.\nabla\right)\overline{H} - \overline{H}\left(\nabla.\overline{v}\right)$$

In this case the rate of decay of magnetic field is much less therefore this is the case of high conductivity.

## EQUATION OF ENERGY

The law of conservation of energy leads to another equation of fluid flow the energy equation. The law of conservation of energy is equivalent to the first law of thermodynamics which states that if a small quantity of heat is added to a simple system it is used up in changing the internal energy of the system and in the external work done by the system. Mathematically this law can be expressed as,

$$d\varepsilon = dQ - pd\left(\frac{1}{\rho}\right)$$

Where dQ is the heat added per unit mass, $d\varepsilon$ is the increase in internal energy per unit mass and $pd\left(\frac{1}{\rho}\right)$ is the amount of work done during the change of volume $d\left(\frac{1}{\rho}\right)$. The heat dQ is obtained by heat conduction.

For a perfect gas, the molecules have very negligible volume and there are no mutual attractions between the individual molecules and hence no potential energy.

Then energy per unit volume of fluid flow is the sum of the kinetic energy density due to translatory motion of the molecules $\frac{1}{2}\ \rho\ v^2$ and the internal energy density $\rho\varepsilon$.

The equation of motion is reduced to,

$$\rho\ \frac{D\overline{v}}{Dt} = -\ \text{grad}\ p + q\overline{E} + \mu\ \overline{J} \times \overline{H},$$ neglecting the gravitational force.

Taking the scalar product of the above equation and the velocity vector $\overline{v}$ we get for the kinetic energy.

$$\rho\ \frac{D}{Dt}\left(\frac{v^2}{2}\right) = -\left(\overline{v}.\nabla\right)p + q\left(\overline{E}.\ \overline{v}\right) + \mu\left[\overline{V}\left(\overline{J}\times\overline{H}\right)\right]$$

The internal energy of the perfect gas depends on its temperature T only.

We know,

$$d\varepsilon = dQ - p\ d\left(\frac{1}{\rho}\right)$$

$$\frac{d\varepsilon}{dt} = \frac{dQ}{dt} - p\ \frac{d}{dt}\left(\frac{1}{\rho}\right)$$

or $$\frac{d\varepsilon}{dt} = \frac{dQ}{dt} + \frac{p}{\rho^2}\ \frac{d\rho}{dt}$$

The equation of continuity for a compressible fluid is,

$$\frac{d\rho}{dt} + \rho\ \text{div}\ \overline{v} = 0$$

So,

$$\frac{d\varepsilon}{dt} = \frac{dQ}{dt} - \frac{p}{\rho}\ \nabla.\ \overline{v}$$

It is an experimental fact that heat flow across an element of fluid surface in $\delta t$ time is proportional to the gradient in the temperature i.e.

$$\delta Q\ \alpha\ -\delta S\ \ \delta t\ \frac{\partial T}{\partial X}$$

or,

$$\delta Q = -k\ \ \delta A\ \ \delta t\ \frac{\partial T}{\partial X}$$

where k is the coefficient of thermal conductivity. The negative sign implies that heat flows from points of higher temperatures to points of lower temperatures.

Consider a closed surface S enclosing an arbitrary volume V. The heat flow out in $\delta t$ time is.

$$-\delta t \iint k \frac{\partial T}{\partial x} \, dS = -\delta t \iint k (\nabla T) \, dS$$

$$= -\delta \iiint \nabla . (k \nabla T) \, dV$$

The heat added to the system per unit volume is then,

$$\rho \delta Q = \delta t \nabla [k \nabla T]$$

Therefore as $\delta t \rightarrow 0$

$$\rho \frac{dQ}{dt} = \nabla . [k \nabla T]$$

So for the internal energy

$$\rho \frac{D\varepsilon}{Dt} = \rho \frac{DQ}{Dt} - p \, (\nabla . \, \bar{v})$$

$$\rho \frac{D\varepsilon}{Dt} = \nabla . [k \nabla T] - p \, (\nabla . \bar{v})$$

Hence the sum of kinetic and internal energy gives

$$\rho \frac{D}{Dt} \left( \varepsilon + \frac{v^2}{2} \right) = - (\bar{v} . \nabla) p - p \, (\nabla . \bar{v}) + \nabla [k \nabla T] + q \, (\bar{E} . \, \bar{v}) + \mu \left[ \bar{v} . \, (\bar{J} \times \bar{H}) \right]$$

$$\rho \frac{D}{Dt} \left( \varepsilon + \frac{v^2}{2} \right) = - \nabla \, (p\bar{v}) + \nabla . [k \nabla T] + q \, (\bar{E} . \, \bar{v}) + \mu \, \left[ \bar{v} . \, (\bar{J} \times \bar{H}) \right] \qquad (1.02)$$

Now we find the ordinary energy equation of an electromagnetic field. When the fluid moves in an electric and magnetic field it experiences a force known as Lorentz force and experimentally its value is

$$\bar{F} = q\bar{E} + \frac{1}{c} (\bar{J} \times \bar{B})$$

Where $q\bar{E}$ is the electric force per unit volume due to an electric field and $(\bar{J} \times \bar{B})$ is the magnetic force per unit volume due to magnetic field.

The rate of work done by the Lorentz force $\overline{F}$ is.

$$\overline{F}.\,\overline{v} = q\left(\overline{E}.\,\overline{v}\right) + \frac{1}{c}\left[\overline{v}.\left(\overline{J} \times \overline{B}\right)\right] \tag{1.03}$$

This force is the cause of the motion of charges if they are free in the field of $\overline{E}$ and $\overline{B}$. If $\overline{v}$ is the velocity of motion of the average density q at any point, we have

$\overline{J} = q\overline{v}$

so,

$$\overline{F}.\,\overline{v} = \overline{E}.\,\overline{J} + \frac{\mu\, q}{c}\left[\left(\overline{v}.\left(\overline{v} \times \overline{H}\right)\right)\right]$$

or

$$\overline{F}.\,\overline{v} = \overline{E}.\,\overline{J} \qquad \text{as } \overline{v}.\left(\overline{v} \times \overline{H}\right) = 0 \tag{1.04}$$

Now we transform the right hand side of above equation (1.04) into a function of $\overline{E}, \overline{D}, \overline{B}$ *and* $\overline{H}$ with the help of Maxwell equations.

Since,

$$\text{Curl } \overline{H} = \frac{4\pi}{c}\overline{J} + \frac{1}{c}\frac{\partial \overline{D}}{\partial t}$$

So,

$$\overline{E}.\,Curl\ \overline{H} = \frac{4\pi}{c}\left(\overline{E}.\,\overline{J}\right) + \frac{1}{c}\left(\overline{E}\,\frac{\partial \overline{D}}{\partial t}\right)$$

Using the vector identity, we have

$$\overline{E}.\ \text{Curl } \overline{H} = \overline{H}.\ \text{Curl } \overline{E} - \text{div}\left(\overline{E} \times \overline{H}\right)$$

with the aid of this result

$$\overline{E}.\,\overline{J} = \frac{c}{4\pi}\left[\overline{H}.\ \text{Curl } \overline{E} - \text{div}\left(\overline{E} \times \overline{H}\right)\right] - \frac{1}{4\pi}\left(\overline{E}.\,\frac{\partial \overline{D}}{\partial t}\right)$$

Since $\text{Curl } \overline{E} = -\dfrac{1}{c}\dfrac{\partial \overline{B}}{\partial t}$

$$\overline{E}.\,\overline{J} = -\frac{c}{4\pi}\text{div}\left(\overline{E} \times \overline{H}\right) - \frac{1}{4\pi}\left[\left(\overline{H}.\,\frac{\partial \overline{B}}{\partial t} + \left(\overline{E}.\,\frac{\partial \overline{D}}{\partial t}\right)\right)\right]$$

If the medium is isotropic then

$$\frac{\partial \overline{H}}{\partial t} = 0, \qquad \frac{\partial \overline{E}}{\partial t} = 0$$

We write,

$$\overline{E}.\,\overline{J} = -\frac{c}{4\pi}\,\mathrm{div}\,\left(\overline{E} \times \overline{H}\right) - \frac{1}{4\pi}\left[\frac{\partial}{\partial t}\left(\overline{H}.\,\overline{B}\right) + \frac{\partial}{\partial t}\left(\overline{E}.\,\overline{D}\right)\right]$$

or

$$\overline{E}.\,\overline{J} = -\frac{c}{4\pi}\,\mathrm{div}\,\left(\overline{E} \times \overline{H}\right) - \frac{\partial}{\partial t}\left(\frac{\mu H^2 + \in E^2}{8\pi}\right)$$

Since a part of the power density is the rate of variation of $\left(\in E^2 - \mu H^2\right)/8\pi$, this quantity must be the sum of energy densities in electric and magnetic fields.

$$\frac{\partial}{\partial t}\left(\frac{\in E^2}{8\pi} + \frac{\mu H^2}{8\pi}\right) = -\left(\frac{c}{4\pi}\,\mathrm{div}\,\left(\overline{E}\ \ \overline{H}\right) + \overline{E}.\ \overline{J}\right) \tag{1.05}$$

Using (1.03) and (1.04) equation (1.02) becomes,

$$\rho\,\frac{D}{Dt}\left(\in + \frac{v^2}{2}\right) = -\,\mathrm{div}\,\left(\overline{v}\,p\right) + \overline{F}.\overline{v} + \mathrm{div}\,\left(k\nabla T\right)$$

$$= \left[-\,\mathrm{div}\,\left(\overline{v}\ \ p\right) + \overline{E}.\,\overline{J} + \mathrm{div}\,\left(k\ \nabla\ T\right)\right] \tag{1.06}$$

Adding equations (1.05) and (1.06) we get

$$\rho\,\frac{D}{Dt}\left(\varepsilon + \frac{v^2}{2}\right) + \frac{\partial}{\partial t}\left(\frac{\in E^2}{8\pi} + \frac{\mu H^2}{8\pi}\right) = -\nabla.\left(\overline{v}\ p\right) + \nabla.\left(k\nabla T\right) - \frac{c}{4\pi}\nabla.\left(\overline{E} \times \overline{H}\right)$$

This is an equation for total energy per unit volume of the flow field when there is no heating by radiant energy.

The terms on the left hand side of above equation give the rate of change of energy per unit volume in the field. The right hand side terms of the above equation have the form of divergence so that, on integrating through a certain volume, all of them reduce to surface integrals.

The first term represents the work done by the pressure force, second term represents the heat loss through conduction from the surface and the last term represents the out flow of electromagnetic energy through the surface where $\overline{E} \times \overline{H}$ is the Poyting vector.

The above equation of energy can be written as,

$$\frac{\partial}{\partial t}\left(\rho\varepsilon+\frac{1}{2}\rho\,v^2\right)+\frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{8\pi}+\frac{\mu H^2}{8\pi}\right)+$$

$$+\,\mathrm{div}\left[\rho\,\bar{v}\left(\varepsilon+\frac{1}{2}v^2\right)+p\bar{v}\right]=\mathrm{div}\left(k\ \mathrm{grad}\ T\right)-\frac{c}{4\pi}\mathrm{div}\left(\bar{E}\times\bar{H}\right)$$

For adiabatic flow no heat is added, conducted or radiated from the flow field. i.e.

$dQ = 0$

The energy equation for the adiabatic flow field

## SHOCK WAVES AND THEIR EXISTENCE

If a small disturbance (i.e. with an infinitesimal amplitude and small velocity) is created within a non-viscous isentropic compressible fluid it will propagate throughout the fluid as a wave motion and with the velocity of sound relative to the fluid without suffering any distortion.

In this case we get the wave equations which shows that both the density and velocity variations follow the same wave patterns derived from the linearizing of the equations of fluid motion. The solution $\rho$ (x,t) and u (x,t) both are the single valued function. The equations of fluid motion reduced to linear form as velocity is small so that the term ($\bar{v}$ . grad) $\bar{v}$ in Euler's equation of motion may be neglected.

If the assumptions of infinitesimal amplitudes and gradients are removed then the wave velocity will not be a constant and a simple wave velocity will distort as it propagates.

In this case as velocity is not small we have to solve the complete non-linear equations of fluid motion. The method of characteristics can be applied to solve such hyperbolic type equations.

A simple analytic solution of these equations of fluid motion may be find such that the density is a function of velocity only. After some manipulation we get equations in u and $\rho$. General solutions of these equations are

$u = f_1\,[x - (u \pm a)\,t]$

$\rho = f_2 [x- (u \pm a) t]$

Where $f_1$ and $f_2$ are arbitrary functions.

We consider the solution,

$u = f_1 [x-(u + a) t]$

This equation shows that the disturbance is propagated at an instantaneous velocity u +a, instantaneous because this velocity is a function of time. If the increment in velocity u is much smaller than speed of sound then the solution is the case of sound wave i.e. the linear case and the curve $u = r_1$ (x-at) does not change its shape as the disturbance propagates. But if u is not small the shape of the wave is disturbed as is propagates.

The resulting distortions in velocity distribution are shown in figure.

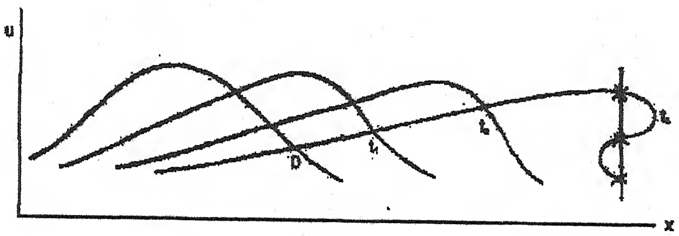


Fig.-6 Formation of Shock

At a time $t_1$ the points of high velocity move faster to the right than those of lower velocity. Thus the creast has moved faster to the right than the trough and so the profile is disturbed. The compressive part of the wave where the propagation velocity is a decreasing function of x, distorts to give a triple valued solution for u (x,t) which is physically impossible for longitudinal wave.

Actually the difficulty is over come by the formation of a 'shock wave'. In such cases a large variation in pressure and density occurs in a very narrow region in which the flow variables change rapidly and the fluid no longer undergoes isentropic changes. The thickness of this shock wave region is very small and so we may considered the shock wave as a surface of discontinuity for many practical problems of inviscid fluids [4], [7].

The propagation of shock is faster than sound when observed from one side of the discontinuity and less than that of sound when observed from the other side. Hence the velocity of the shock will be supersonic viewed from ahead and it will be subsonic viewed from behind.

The formation of these shock waves causes a great noise equal to that of an explosion usually termed as supersonic bang so the effect of shock must taken into account in the design of aeroplanes, pipe flow, supersonic flight of projectiles and so on.

For the flow of an inviscid and non-conducting gas the laws of conservation of mass, momentum and energy are originally formulated in the differential equation form as it is assumed that the flow variables defining the flow are continuous functions. Flows are also possible however for which discontinuities in the distribution of these flow variables occur. So the conservation laws can also be applied to such discontinuous flow and hence across a shock.

The condition for the existence of shock waves which may be called the jump conditions, relate the velocity, pressure, density and temperature in front of the shock to those behind of them. The jump conditions are the simple consequence of the laws of conservation of mass, momentum and energy across the surface of discontinuity and the equation of state of the medium through which the shock is moving.

## JUMP CONDITIONS ACROSS A DISCONTINUITY

A differential equation can be represented as

$$\frac{\partial H}{\partial t} + \frac{\partial F}{\partial X} = K \tag{1.07}$$

Where H, F, K are functions of position and time. K contribute for sources or sinks. F is the flux per unit time and H is the distribution of some state of medium. Functions H, F and K are continuous and differentiable.

Let $x_1(t)$ and $x_2(t)$ are continuous and differentiable function of t and $x_1(t) < x_2(t)$ for every t.

Integrating (1.7),

$$\int_{x_1(t)}^{x_2(t)} \frac{\partial H}{\partial t} \, dx + F(x_2) - F(x_1) = \int_{x_1(t)}^{x_2(t)} K \, dx \tag{1.08}$$

Now we know that Newton-Leibnitz formula is

$$\frac{\partial}{\partial t} \int_{x_1(t)}^{x_2(t)} H(x,t)\, dx = \int_{x_1}^{x_2(t)} \frac{\partial H}{\partial t} \, dx + H(x_2,t) \frac{dx_2}{dt} - H(x_1,t) \frac{dx_1}{dt}$$

Using this formula (1.08) becomes

$$\int_{x_1(t)}^{x_2(t)} \frac{\partial u}{\partial t} \, dx + H(x_2,t) \frac{dx_2}{dt} - H(x_1,t) \frac{dx_1}{dt} = F(x_2) - F(x_1) + \int_{x_1(t)}^{x_2(t)} K \, dx$$

If $x_1$ and $x_2$ are constants then

$$\frac{\partial}{\partial t} \int_{x_1}^{x_2} H \, dx = F(x_2) - F(x_1) + \int_{x_1}^{x_2} K \, dx \tag{1.09}$$

Where $F(x_1)$ is the outgoing flux and $F(x_1)$ is the incoming flux.

The above result (1.09) states that,

The time rate of change of total amount of state of the medium in any section $x_1 < x < x_2 \left( \text{i.e. L.H.S.} \frac{\partial}{\partial t} \int_{x_1}^{x_2} H \, dx \right)$ is equal to the difference of outgoing flux and incoming flux and some contribution of sources and sinks.

If there is no sources or sinks i.e. $K = 0$ and,

$H = H(U)$, $F = F(U)$

then (1.09) represents the conservation of mass.

$$\frac{\partial}{\partial t} \int_{x_1}^{x_2} H \, dx = F(x_2) - F(x_1)$$

and equation (1.07) is in the conservation form,

$$\frac{\partial H}{\partial t} + \frac{\partial F}{\partial x} = 0$$

The most general conservation form for differential equation is

$$\frac{\partial}{\partial t}\left(U^n\right) + \frac{\partial}{\partial t}\left(\frac{n}{n+1} U^{n+1}\right) = 0, \ n = 1,2.... \qquad (1.10)$$

Suppose there is a discontinuity at x = X (t) and $x_1$ and $x_2$ are chosen so that $x_1 < X(t) < x_2$. Suppose $U^n$ and $U^{n+1}$ and their first derivatives are continuous in X (t) $> x \geq x_1$ and in $x_2 \geq x >$ X (t) and have finite limits as $x \to X(t)$ from below and above.

Integrating (1.10).

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t} U^n \, dx = \frac{n}{n+1}\left\{U^{n+1}(x_1) - U^{n+1}(x_2)\right\} \qquad (1.11)$$

Also using the property of line integral,

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t} \, dx = \int_{x_1}^{x(t)} \frac{\partial u^n}{\partial t} dx + \int_{x(t)}^{x_2} \frac{\partial U^n}{\partial t} dx$$

Now we use the Newton-Leibnitz formula on the left hand side

$$\int_{x_1}^{x_2} \frac{\partial U^n}{\partial t} dx = \frac{\partial}{\partial t}\int_{x_1}^{x(t)} U^n \, dx + U^n(\bar{x}, t)\frac{dx}{dt} + \frac{\partial}{\partial t}\int_{x(t)}^{x_2} U^n dx - U^n(x^t, t)\frac{dx}{dt}$$

Where $U^n(X^-, t)$, $U^n(X^+, t)$ are the value of $U^n(x,t)$ as $x \to X(t)$ from left and right respectively.

$$\int_{x_1}^{x_2} \frac{\partial U^n}{\partial t} dx = \frac{\partial}{\partial t}\int_{x_1}^{x(t)} U^n \, dx + \frac{\partial}{\partial t}\int_{x(t)}^{x_2} U^n \, dx + S\left(U_i^n - U_r^n\right) \qquad (1.12)$$

where $\quad S = \dfrac{dx}{dt}$

and $\quad U^n(X^-, t) \rightarrow U_l^n \,,\; U^n(X^+, t) \rightarrow U_r^n$

On comparing (1.11) and (1.12) we get,

$$\frac{\partial}{\partial t}\int_{x_1}^{x(t)} U^n\,dx + \frac{\partial}{\partial t}\int_{x(t)}^{x_2} U^n\,dx + S\left(U_l^n - U_r^n\right) = \frac{n}{n+1}\left[U^{n+1}(x_1) - U^{n+1}(x_2)^2\right]$$

Since $\dfrac{\partial u^n}{\partial t}$ is bounded in each of the intervals separately the integral tends to zero in the limit as $x_1 \rightarrow x^-$ and $x_2 \rightarrow x^+$. Therefore,

$$S\left(U_l^n - U_r^n\right) = \frac{n}{n+1}\left[U^{n+1}(x_1) - U^{n+1}(x_2)\right]$$

$$= \frac{n}{n+1}\left[U^{n+1}(x^-) - U^{n+1}(x^2)\right]$$

or $\quad S\left(U_l^n - U_r^n\right) = \dfrac{n}{n+1}\left(U_l^{n+1} - U_r^{n+1}\right)$

This condition may also be written as

$$S\left[U^n\right] = \frac{n}{n+1}\left[U^{n+1}\right]$$

or $\quad -S\left[U^n\right] + \dfrac{n}{n+1}\left[U^{n+1}\right] = 0$

where the brackets [ ] indicate the jump in quantity and S is the speed of propagation of the discontinuity or shock front.

$$S = \frac{dx}{dt} = \frac{n}{n+1}\left[\frac{U^{n+1}}{U^n}\right]$$

This give the location of the discontinuity. So if the conservation equation is

$$\frac{\partial H}{\partial t} + \frac{\partial F}{\partial x} = 0 \tag{1.13}$$

then the jump relation is [7]

$$-S\,[H] + [F] = 0 \tag{1.14}$$

## JUMP CONDITIONS IN ORDINARY FLUIDS

Consider a stationary shock front (discontinuity) separating two regions i.e. $S = \frac{dx}{dt} = 0$ so the jump condition (1.14) becomes,

$$[F] = 0 \tag{1.15}$$

The equation representing the conservation of mass, momentum and energy across a surface of discontinuity are given below

Equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i\right) = 0$$

which is in the form (1.13). So the corresponding jump condition (1.15) is

$$\left[\rho v_i n_i\right] = 0$$

*i.e.* $\rho_1 v_{1i} n_i = \rho_2\ v_{2i}\ n_i = m$

Equation of energy is

or $$\frac{\partial}{\partial t}\left(\rho v_j\right) - v_j \frac{\partial \rho}{\partial t} + \rho v_i \frac{\partial}{\partial x_i} v_j = -\frac{\partial}{\partial x_i} p$$

or $$\frac{\partial}{\partial t}\left(\rho v_j\right) - v_j \frac{\partial \rho}{\partial x_i}\left(\rho v_i\right) + \rho v_i \frac{\partial}{\partial x_i} v_j = -\frac{\partial}{\partial x_i} p$$

(using equation of continuity)

or $$\frac{\partial}{\partial t}\left(\rho v_j\right) - v_j\frac{\partial v_i}{\partial x_i} + v_j v_i\frac{\partial \rho}{\partial x_i} + \rho v_i\frac{\partial}{\partial x_i}v_j = -\frac{\partial}{\partial x_i}p$$

or $$\frac{\partial}{\partial t}\left(\rho v_j\right) + \frac{\partial}{\partial x_i}\left(\rho v_i\, v_j\right) = -\frac{\partial}{\partial x_i}p$$

$$\frac{\partial}{\partial t}\left(\rho v_j\right) + \frac{\partial}{\partial x_i}\left(\rho v_i\, v_j + p\right) = 0$$

which is in the conservation form, jump condition is

$$\left[\left(\rho v_i\, v_j + p\right)n_i\right] = 0$$

using (1.16) a we have,

$$[p]\,n_i = -m\left[v_j\right] \qquad (1.16)b$$

Equation of energy is

$$\frac{dh}{dt} = \frac{1}{\rho}\frac{dp}{dt}$$

In conservation form it can be written as

$$\frac{\partial}{\partial t}\left(\rho F + \frac{\rho v^2}{2}\right) + \frac{\partial}{\partial n_i}\left[\rho v\left(\frac{p}{\rho} + f + \frac{v^2}{2}\right)\right] = 0$$

where $h = \varepsilon + \dfrac{p}{\rho}$

and $\varepsilon = c_v T$

$$\varepsilon = \frac{c_v p}{\rho R} = \frac{p}{\rho}\left(\frac{c_v}{c_p - c_v}\right)$$

$$= \frac{P}{\rho(\gamma - 1)}$$

So

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1}+\frac{\rho v^2}{2}\right)+\frac{\partial}{\partial x_i}\left\{\rho v_i\left(\frac{\gamma p}{\rho(\gamma-1)}+\frac{v^2}{2}\right)\right\}=0$$

Jump condition is

$$\left[\rho v_i\left(\frac{\gamma p}{\rho(\gamma-1)}+\frac{v^2}{2}\right)n_i\right]=0$$

or $$\left[\rho v_i n_i\left(\frac{\gamma p}{\rho(\gamma-1)}+\frac{v^2}{2}\right)\right]=0$$

If $\rho_1 \; v_{1i}\, n_i = \rho_2\, U_{2i}\; n_i$ then the constant factor may be dropped in the above condition and we have

$$\left[\frac{\gamma p}{\rho\,(\gamma-1)}+\frac{v^2}{2}\right]=0 \tag{1.16)c}$$

The equations [1.16] are the Runkine Hugoniot jump conditions when the shock is at rest The subscript 1and 2 denote a variable ahead and behind of the shock front are the components of the unit normal to the shock and bracket denotes the difference of values in the two sides of the shock surface of the quantity enclosed.

**JUMP CONDITIION IN MAGNETOGASDYNAMICS**

In an electrically conducting fluids in the presence of magnetic field, discontinuity (i.e a shock wave) in flow variables can exists. The study of magnetohydrodynamic shock waves was begun in 1950 with the paper of F. de Hoffmann and Taylor [9] .since then continued interest inspired by astrophysics by flight at the outer edges of atmosphere etc. has produced many papers describing shock wave properties. The basic properties of magnetogasdynamic shock waves are determined by the conservation laws. The presence of magnetic field the relations connection the flow and field quantities on the two sides of the shock surface are as follow [10] , Equation of continuity

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho v_i\right)=0$$

which is in the form (1.13) so the corresponding jump condition (1.15) is, $[pv_i\, n_i] = 0$

or $[\rho vn] = 0$

i.e. $\rho_1 vn_1 = \rho_2 vn_2$ (1.17)a

The equation of motion is,

$$\frac{\partial}{\partial t}(\rho u_j) + u_j \frac{\partial}{\partial x_i}(\rho u_i) + \rho\left(u_i \frac{\partial}{\partial x_i}\right) u_j = -\frac{\partial}{\partial x_i}\left(\rho + \frac{\mu H^2}{8\pi}\right)\delta_{ij} + \frac{\mu}{4\pi}\left(H_i \frac{\partial}{\partial x_i}\right) h_j$$

since $$\frac{\partial}{\partial x_i}(\rho v_i\, v_i) = v_j \frac{\partial}{\partial x_i}(\rho v_i) + \rho v_i \frac{\partial}{\partial x_i} v_j$$

and $$\frac{\partial}{\partial x_i}(H_i\, H_j) = H_j \frac{\partial}{\partial x_i} H_i + H_i \frac{\partial}{\partial x_i} H_j$$

so, $$H_i \frac{\partial}{\partial x_i} H_j = \frac{\partial}{\partial x_i}(H_i\, H_j) - H_j \frac{\partial}{\partial x_i} H_i = \frac{\partial}{\partial x_i}(H_i\, H_j)$$

as $\operatorname{div} \overline{H} = 0$

Using these results equation of motion can be written as

$$\frac{\partial}{\partial t}(\rho v_j) + \frac{\partial}{\partial x_i}(\rho v_i\, v_j) = -\frac{\partial}{\partial x_i}\left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij} + \frac{\mu}{4\pi}\frac{\partial}{\partial x_i}(H_i\, H_j)$$

or $$\frac{\partial}{\partial t}(\rho v_j) + \frac{\partial}{\partial x_i}\left(\rho v_i\, v_j - \frac{\mu}{4\pi} H_i\, H_j + \left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij}\right) = 0$$

which is in the conservation form. So the jump condition is,

$$\left[\left(\rho v_i\, vj_j - \frac{\mu}{4\pi} H_i\, H_j + \left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij}\right)\eta_i\right] = 0$$

or $$\left[\left(p + \frac{\mu H^2}{8\pi}\right)\overline{\eta} + \rho\,\overline{v} u_\eta - \frac{\mu}{4\pi}\overline{H}\, H_\eta\right] = 0$$

or $$[\rho\, \overline{v} v_\eta] + \left[\left(p + \frac{\mu H^2}{8\pi}\right)\overline{\eta}\right] = \frac{\mu}{4\pi}[\overline{H}\, H_\eta]$$ (1.17)b

Also we know

$$\text{div } \overline{H} = 0$$

i.e. $$\frac{\partial}{\partial x_i} H_i = 0$$

Jump condition is,

$$[H_i \, n_i] = 0$$

or

$$[Hn] = 0 \tag{1.17)c$$

Equation for the variation of the magnetic field (when $\sigma \to \infty$) is

$$\frac{\partial H_j}{\partial t} = H_i \frac{\partial}{\partial x_i} v_j - v_i \frac{\partial}{\partial x_i} H_j - H_j \frac{\partial}{\partial x_i} v_i$$

or $$\frac{\partial H_j}{\partial t} = \frac{\partial}{\partial x_i}\left(H_i \, v_j - v_j \frac{\partial H_i}{\partial x_i} - \frac{\partial}{\partial x_i}(v_i \, H_j) + H_j \frac{\partial v_i}{\partial x_i} - H_j \frac{\partial}{\partial x_i} v_i\right)$$

or $$\frac{\partial H_j}{\partial t} = \frac{\partial}{\partial x_i}(H_i \, v_j - v_i \, H_j)$$

or $$\frac{\partial H_j}{\partial t} + \frac{\partial}{\partial x_i}(v_j H_j - H_i \, v_j) = 0$$

Jump condition is

$$[(v_i H_j - H_i v_j) n_i] = 0 \tag{1.17)d$$

The equation of energy is

$$\frac{\partial}{\partial t}\left\{\rho\left(\varepsilon + \frac{1}{2} v^2 + \frac{H^2}{8\pi}\right)\right\} + \nabla\left\{\overline{v}\left(p + \rho\varepsilon + \frac{1}{2}\rho v^2\right) + \frac{c}{4\pi}\, \overline{\varepsilon} \times \overline{H}\right\} = 0$$

Ohm's law for a moving medium is

$$\bar{J} = \sigma\left(\bar{\varepsilon} + \frac{\mu}{c}\,\bar{v} \times \bar{H}\right)$$

or $$\bar{\varepsilon} = \frac{1}{\sigma}\bar{J} - \frac{\mu}{c}\,\bar{v} \times \bar{H}$$

$$\therefore \quad \frac{c}{4\pi}\,\bar{\varepsilon} \times \bar{H} = \frac{\mu}{4\pi}\,\bar{H} \times \left(\bar{v} \times \bar{H}\right) - \frac{c}{4\pi\sigma}\,\bar{H} \times \bar{J}$$

also we know

$$\text{Curl}\,\bar{H} = \frac{4\pi}{c}\,\bar{J}$$ (neglecting the displacement current)

$$\frac{c}{4\pi}\,\bar{\varepsilon} \times \bar{H} = \frac{\mu}{4\pi}\,\bar{H} \times \left(\bar{v} \times \bar{H}\right) - \frac{c^2}{16\pi^2\sigma}\,\bar{H} \times \left(\nabla \times \bar{H}\right)$$

In the case of high conductivity it reduces to

$$\frac{c}{4\pi}\,\bar{\varepsilon} \times \bar{H} = \frac{\mu}{4\pi}\,\bar{H} \times \left(\bar{v} \times \bar{H}\right)$$

$$= \frac{\mu}{4\pi}\left(H^2\bar{v} - \left(\bar{H}\ \bar{v}\right)\bar{H}\right)$$

using this result energy equation can be written as

$$\frac{\partial}{\partial t}\left\{\rho\left(\varepsilon + \frac{1}{2}v^2 + \frac{H^2}{8\pi}\right)\right. + \nabla\left\{\bar{v}\left(p + \rho\varepsilon + \frac{1}{2}\rho v^2\right) + \frac{\mu}{4\pi}\left(H^2\bar{v} - \left(\bar{H}.\ \bar{v}\right)\bar{H}\right)\right\}\Bigg\} = 0$$

or

$$\frac{\partial}{\partial t}\left\{\rho\left(\varepsilon + \frac{1}{2}v^2 + \frac{H^2}{8\pi}\right)\right\} + \frac{\partial}{\partial x_i}\left\{v_i\left(p + \rho\varepsilon + \frac{1}{2}\rho v^2 + \frac{\mu H^2}{4\pi} - \frac{\mu}{4\pi}\left(H_j\ v_j\right)H_i\right)\right\} = 0$$

Jump condition is

$$\left[\left\{v_i\left(p + \rho\varepsilon + \frac{1}{2}\rho v^2 + \frac{\mu H^2}{4\pi}\right) - \frac{\mu}{4\pi}\left(H_j.\ v_j\right)H_i\right\}H_i\right] = 0$$

or

$$\left[ v_n \left( \frac{1}{2} \rho v^2 + \rho\varepsilon + \frac{\mu H^2}{4\pi} + p \right) \right] = \frac{\mu}{4\pi} \left[ \left( \bar{v}.\, \bar{H} \right) H_n \right] \tag{1.17)e$$

The equation (1.17) are the shock conditions in the frame of reference in which shock is at rest. The equations (1.17) are also called as the MHD Rankine-Hugoniot relation, which can be written as from equations (1.17)

$$\frac{p_1}{p_2} = \frac{(\gamma+1)\rho_1 - (\gamma-1)\rho_2}{(\gamma+1)\rho_2 - (\gamma-1)\rho_1} + \frac{\mu [H]^2 (\gamma-1)(\rho_1 - \rho_2)}{8\pi p_2 \{(\gamma+1)\rho_2 - (\gamma-1)\rho_1\}}$$

or

$$\frac{\rho_1}{\rho_2} = \frac{(\gamma+1)p_1 + (\gamma-1)p_2}{(\gamma-1)p_1 + (\gamma+1)p_2} - \frac{\mu [H]^2 (\gamma-1)(\rho_1 - \rho_2)}{8\pi \rho_2 \{(\gamma-1)p_1 + (\gamma+1)p_2\}}$$

For $\bar{H} = 0$ this relation gives the result for a non conducting fluid.

The usual situation is that the flow ahead of the shock is known and these conditions are used to determine the flow behind or to determine the flow quantities in terms of one of the flow quantity behind.

**RADIATION PHENOMENON**

At very high temperature radiation can be considered as a continuous emission of energy in the form of electromagnetic wave which propagate in vacuum with the velocity of light. This energy is called as radiant energy of thermal radiation. Whereas, according to "quantum theory" the radiant energy emitted or absorbed is not continuous permitting all possible values, as demanded by the wave theory but in a discrete quantified form as integral multiples of an elementary quantum of energy, photon or light quanta. The amount of energy in each quantum being given by the product $h\upsilon$ where $\Omega$ is planck's constant and $\upsilon$ and frequency of the radiation.

Thus the quantum theory proposes the particle characteristics of radiation while the classical theory the wave characteristics, both being required to understand the complex behaviour of radiation. It has also been recognised that radiation reveals itself in

different types such as the electrical (radio) waves, infrared, visible, ultraviolet, X-rays and gamma rays. This theory of thermal radiation can be applied to understand the processes which take place in stellar media, to explain the observed luminosity of stars and nuclear explosions and also to high temperature fluid flow.

Radiative transfer and radiative heat exchange have an influence on both the state and the motion of the fluid. This influence is caused by the fact that fluid loses or gains energy by emitting or absorbing heat. The state of the fluid can be described by the fundamental equations which, in the presence of radiation field must include the interaction between the radiation and the fluid. There are three different thermal radiation effects on the flow field ([11], [12], [13])

## RADIANT ENERGY DENSITY $E_r$

The radiant energy density per unit mass of the fluid is given by,

$$\frac{E_r}{\rho} = \frac{aT^4}{\rho}$$

Where a is stefan-Boltzman constant.

## RADIANT PRESSURE $P_r$

According to the well known result of classical electrodynamics the pressure of a radiation field is equal to the one-third of the radiant energy density i.e

$$P_r = \frac{1}{3}E_r$$

$$= \frac{1}{3}aT^4$$

The radiant energy density and radiant pressure become comparable with the energy density and pressure of the fluid only at extremely high temperature or extremely low gas densities.

**RADIATION FLUX $F_r$**

The net amount of radiant energy passing through the surface per unit area per unit time called the radiant flux through the surface and is given by,

$$F_r = \frac{C}{4}\,\frac{aT^4}{\rho}$$

Where C is the velocity of light in vacuum.

The amount of energy lost or, conversely, the energy released in the fluid as the interaction with the radiation is given by the divergence of the radiation flux. Thus a comparison of radiant energy flux and the flux of energy of the fluid can be used to characterize the importance of the radiant heat transfer in the medium.

Let, q be the energy lost by radiation in a unit volume of the fluid per unit time. then,

$$q = \nabla.\overline{F}_r$$

If the fluid radiates more energy then it absorbs, then the energy is lost in radiation and q>0; if more energy is absorbed than emitted, then the fluid is heated by radiation and the energy loss is negative, q<0.

Neglecting the effect of viscosity and heat conductivity, the fundamental equations for the fluid-radiation systems can be written as follow,

The continuity equation remains unchanged

$$\frac{\partial\rho}{\partial t} + div\left(\rho\overline{v}\right) = 0$$

The equations of motion taking radiant pressure into account and neglecting the external force are

$$\rho\,\frac{D\overline{v}}{Dt} = -grad\ (p + p_r)$$

In the energy equation the radiant energy density $E_r$ must be added to the energy density of the fluid and also this equation requires the introduction of a term describing the energy losses by radiation. The energy equation (for adiabatic flow) then becomes

$$\rho \frac{D}{Dt}\left(\varepsilon + \frac{1}{2}v^2 + \frac{E_r}{\rho}\right) = -\, div\,(p\bar{v}) - q$$

replacing q by the divergence of the flux $F_r$

$$\rho \frac{D}{Dt}\left(\varepsilon + \frac{1}{2}v^2 + \frac{E_r}{\rho}\right) = -\, div\,(p\bar{v} + \bar{F}_r)$$

**SPHERICAL AND CYLINDRICAL SHOCK WAVE**

Consider the propagation of a shock wave through a perfect gas of great intensity (i.e. very strong) resulting from a strong explosion i.e. from the instantaneous release of a large quantity of energy.

When the energy is suddenly released in an infinitely concentrated form and distribution of density, pressure etc. depends only on the distance from some point then this is the case of spherical shock wave.

When the energy is suddenly released along a line and distribution of all quantities is homogeneous in some direction and has complete axial symmetry about that direction, then this is the case of cylindrical shock.

Since we consider the symmetrical flow (centrally or axially) so there are only two independent variables, namely r and t. So the one dimensional fundamental equations governing the adiabatic flow are [5], [14]

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + j\frac{u}{r}\right) = 0 \tag{1.18}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \tag{1.19}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\frac{p}{\rho^\gamma} = 0 \tag{1.20}$$

or,

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0$$

Where $j = 0$, 1, or 2 stands for plane, cylindrical or spherical case, respectively, and r is the radial distance from the centre in spherical case, the radial distance from the line of explosion, in the cylindrical case u is radial velocity and $\rho$, the density.

Similarity principle may be used to reduce these equations to ordinary differential equations.

## SIMILARITY PRINCIPLE AND SELF-SIMILAR GAS MOTION

It is not always possible to solve non-linear differential equations describing the physics of a motion or a process by using mathematical techniques. An approximate solution of such a problem can be obtained by solving a similar problem which is easier to solve.

We consider the problem of one-dimensional adiabatic flows of a perfect gas with constant specific heat with either planer, cylindrical or spherical symmetry. The system of equations for flow of this type is given by (1.18) to (1.20). These gas dynamic equations contain five dimensional quantities p, $\rho$, u, r and t. The dimensions of three of which are independent; for example density and time, so the equations admit three independent similarity transformation groups of quantities. Dimensional analysis can be used to obtain these groupings. By the successive application of these three groups we can obtain solutions for the different flow similar to each other with altered density, length and time scales. Analogous transformations are made at the same time in the initial and boundary conditions of the problem.

The motion itself may be described by the most general functions of the two variables r and t, $\rho(r,t)$, $p(r,t)$ and u (r, t). These functions also contain the parameters entering the initial and boundary conditions of the problem. They do not depend upon the position r and time t independently but are functions only of the combination (r/t). In other words the distribution of all quantities with respect to r change with time without changing their form they remain similar to themselves. This type of motion in which the distribution of the flow variables remain similar to themselves (i.e. similarity in the motion itself) with time and vary only as a result of changes in scale is called self similar.

Consider the distribution of pressure. The function p (r,t) can be written in the form p (r,t) = $\pi(t)\,P\,(r/R)$, where $\pi(t)$ is the scale of the pressure and R (t) is the length scale both depend on time in some manner and the dimensionless ratio $\frac{p}{\pi} = P\,(r/R)$ is a function of new dimensionless coordinate $\eta = r/R$. Multiplying the variables P and $\eta$ by the scale function $\pi(t)$ and R (t), we can obtain from the function P ( $\eta$ ), independent of time the true pressure distribution. The other flow variables, density and velocity, are expressed similarly [15].

It is a natural question that what requirement must be satisfied by the conditions of a problem in order that the motion be self-similar. Dimensional analysis is used to answer this question. Since the dimensions of pressure and density contain the unit of mass at least one of the parameters in the problem must also contain a unit of mass. In many cases this is the constant initial density of the gas $\rho_0$ which has the dimension $ML^{-3}$. Let the parameter containing the unit of mass is a. It can be assumed that its dimensions are $[a] = ML^k\ T^s$. The dimensions of the functions p, $\rho$, and u are, $[p] = M\,LT^{-2}$, $[\rho] = M\,L^{-3}$, and $[u]=LT^{-1}$, we can without any loss of generality, represent them in the form suggested by Sedov [16].

$$p = \frac{a}{r^{k+1}\ t^{s+2}}\,P, \qquad \rho = \frac{a}{r^{k+3}\ t^{s}}\,G, \qquad u = \frac{r}{t}\ V$$

Where P, G and V are dimensionless functions that depend on dimensionless groups containing r, t and the parameters of the problem.

For self similar motion it is possible to reduce a system of partial differential equations to a system of ordinary differential equations for new reduced functions of the similarity variables $\eta = r/R, R = R\,(t)$. The boundary and initial conditions of the problem are made dimensionless and in term transformed into conditions on the new unknown function of $\eta$. This simplifies the problem greatly from the mathematical stand point and in a number of cases makes it possible to find exact analytic solutions.

The problem of a strong explosion represents a typical example of a self-similar motion. This problem was formulated and solved by Sedov [16] and succeeded in finding an exact analytic solution to the equations of self-similar motion. The same problem was also considered by Stanyukovinch in his dissertation [17] and by Taylor [18] both of whom formulated the equations for the problem and obtained numerical and not analytic solutions.

The parameters in the problem of a strong explosion are the initial density of the gas $\rho_0 \sim ML^{-3}$ and the energy of explosion $\in\sim ML^2\,T^{-2}$. The energy E is always equal to the total energy of the moving gas and as a result an energy integral appears in the problem. These two parameters can not be combined to yield scaler with dimensions of either length or time. Hence the motion will be self-similar, that is will be a function of a particular combination of the coordinate r (distance from the center of the explosion) and the time t. The initial pressure and speed of sound p and c in the problem of a strong explosion are assumed to be equal to zero and hence these quantities are not parameters of the problem. So the quantity r/t can not serve as the similarity variables. In this case the only dimensional combination which contains only length and time is the ratio of E to $\rho_0$, with the dimension $\left[E/\rho_0\right] = L^5\,T^{-2}$. Hence the dimensionless quantity

$$\eta = r\left(\frac{\rho_0}{\varepsilon\, t^2}\right)^{1/5}$$

can serve as the similarity variable. The distribution of pressure, density, and gas velocity can be expressed as functions of one dimensionless variable $\eta$.

## CONCEPT OF SELF GRAVITATION

A fluid can be referred as self-gravitating when the fluid mass included is large and isolated and the isolated and the gravitational attraction of other parts of the fluid provides the volume force on any particular fluid element for example, a gaseous star.

In the case of spherical symmetry the effect of all masses on a particle (point mass) at a distance r from the centre of symmetry is equal to the force of attraction by the point mass placed at the centre of symmetry and having a mass m (r, t).

The behaviour of gravitationally interacting gaseous masses forming a star is given by the appropriate equations of motion ([12], [16]).

The continuity equation for radial gas motion with spherical symmetry is

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0$$

For the inviscid gas, momentum equation taking gravitational forces into account (according to Newton's law of gravitation gravity g at r is $g = \frac{GM}{r^2}$ can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{GM}{r^2} = 0$$

Where G is the gravitational constant and m (r,t) is the mas sog as with in the sphere of radius r.

To determine mass (m) we use the equation,

$$\frac{\partial m}{\partial r} = 4\pi\rho r^2$$

which expresses the fact that the mass of a spherical shell of radius r and thickness $\partial r$ is $4\pi r^2\rho\,\partial r$ and the energy equation for adiabatic gas motion

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + u\,\frac{\partial}{\partial r}\left(\frac{p}{\rho^\gamma}\right) = 0$$

A very large no. of papers have been published by now in which analogous self-similar solution were obtained to explain the adiabatic unsteady flow in self-gravitating gas and analysed for systems of partial differential equations encountered in various problems of astrophysics such as internal motion in stars.

## EQUATIONS OF MOTION AND JUMP CONDITION IN MAGNETO HYDRODYNAMICS

In an electrically conducting fluids in the presence of magnetic field discontinuity, i.e. Shock wave in flow variable can exit's. The study of magnetohydrodynamic shock waves was begun in 1950 with the paper of F.de Hoffmann and Taylor .

When electric current induced in the fluid, then their flow in the magnetic field produces mechanical forces which modify the motion magnetogas dynamic owes its peculiar interest and difficulty to this interaction between the field and the motion. Thus the equations of magnetogasdynamic are the ordinary and electromagnetic equations. We only take into account the interaction between the motion and the magnetic field, we have ignored the Maxwell's displacement currents. Through out the thesis we take $\mu = 1$ because magnetic permeability $\mu$ differs only slightly from unity which is unimportant. We also assume that the dissipative mechanism such as viscosity, thermal conductivity and electrical resistant are absent.

Since the problems dealt in this thesis relate to magneto gas dynamic or magneto-radiative shocks. We refer in this article the relevant flow and fluid equation.

$$p = \rho RT \; ; \tag{1.18}$$

$$\frac{D\rho}{Dt} + \rho\frac{\partial u}{\partial r} + j\frac{\rho u}{r} = 0 \; ; \tag{1.19}$$

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\upsilon h^2}{\rho r} = 0 \; ; \tag{1.20}$$

$$\frac{Dh}{Dt} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \upsilon\frac{hu}{r} = 0; \tag{1.21}$$

$$\frac{\partial}{\partial t}\left(P\rho^{-\gamma}\right) + u\frac{\partial}{\partial r}\left(P\rho^{-\gamma}\right) = 0 : \tag{1.22}$$

Where $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + u\,\dfrac{\partial}{\partial r}$

and u,p, $\rho$ ,h,r and t are the velocity, pressure, density magnetic field transverse to the flow, radial distance and time respectively, T is the ratio of specific heat at constant volume and constant pressure, J=0, 1 and 2 corresponding to plane. Cylindrical and

spherical respectively and $\upsilon=0$ for plane and $\upsilon=1$ corresponds to cylindrical and spherical both.

In the presence of magnetic field the relation connecting the flow the field quantities on the two sides of the shock surface are as follows (34), (35) where the velocity in front the shock wave is zero :

$$h_2\ (u-u_2) = h_1 u \tag{1.23}$$

$$\rho_2\ (u-u_2)\ = \rho_1\ u \tag{1.24}$$

$$\rho_2 + \frac{1}{2} h_2^2 + \rho_2\,(u-u)_2^2 = \rho_1 + \frac{1}{2} h_1^2 + \rho_1\, u^2 \tag{1.25}$$

$$\frac{1}{2}(u-u_2)^2 + \frac{T\rho_2}{(T-1)\rho_2} + \frac{h_2^2}{\rho_2} = \frac{1}{2}u^2 + \frac{T\rho_1}{(T-1)\rho_1} + \frac{h_1^2}{\rho_1} \tag{1.26}$$

Where sufix 1 and 2 correspond to the value of the quantities just ahead and just behind the shock surface and u is the shock velocity.

## MHD AND FLOW STABILITY

A slow movement of a fluid may be characterized by streamlines indicative of regular, orderly motion, called laminar flow. As the flow velocity is increased or the space available made larger there is a point where any small disturbance is amplified and the flow breaks up into turbulence. Then the velocity profile, and all the other flow characteristics change radically. This is especially familiar in pipe flow, in the transition from laminar Poiseuille flow to turbulent Fanning-Hagen flow. The criterion is the dimensionless Reynolds number $R = VL\rho/\mu$, which indicates the relative importance of inertial and viscous forces. For flow in rough pipes, the transition occurs around R = 2500.

A magnetic field may be expected to stabilize a flow against the transition to turbulent flow. The cases of the magnetic field in the direction of flow and normal to the flow have been studied, and the stabilizing effect observed. The critical Reynolds number is increased as the magnetic Reynolds number is increased. Experiments have been made that show this general result, but they are difficult and quantitatively not very conclusive.

Another kind of instability occurs when there is a temperature gradient and a pressure gradient in the same direction such as occurs in the lower atmosphere or in a pot of liquid heated on a stove. If we imagine a small parcel of fluid from one level suddenly moved to a level where the pressure is different, it will expand or contract adiabatically (to a good approximation) until the pressures are equalized, and its temperature will change by a definite amount. The change in temperature as a function of difference of position defines the adiabatic lapse rate, to use meterorological terminology. If the actual lapse rate (temperature gradient) is greater, then if the parcel has risen, it will find itself hotter and less dense than its surroundings. Buoyancy will then encourage it to rise even higher. Should the parcel have sunk, then things are exactly reversed. and it will find itself cooler and denser, so it will be impelled to sink further. A parcel at 1 just arrived from 0 is hotter than its environment 1'. while a parcel at 2 is cooler than its environment at 2'. If, on the other hand, the lapse rate is less than adiabatic, then a displaced parcel will be encouraged to return to its original position. In the former case, we have convective instability, and in the latter, convectil'e stohilil1". Indeed, the lower part of the atmosphere, in which the decrease in temperature with height is usually more rapid than the adiabatic decrease. and which is. therefore. convectively unstable, is called the troposphere ("sphere of turning"). The unusual opposite case is called an inversion. Convection is important not only in the troposphere and in pots, but also deep in the earth. and in the sun. where energy is struggling to get out [19-20].

A magnetic field in the direction of the temperature and pressure gradient will hinder the transverse motion essential to convection, and make the convective "cells" narrower and less efficient, reducing the rate of energy transfer. In the Sun. this is seen by the relative darkness of sunspots. where a vertical magnetic field of thousands of gauss reduces the efficiency of convection of heat from below, so the surface cools below the general level of the photosphere.

**MHD AND THE SUN**

It was noted above that the only conducting fluids available for laboratory experiments are mercury and liquid sodium, both inconvenient for different reasons. It is

very difficult to reach large magnetic Reynolds numbers in laboratory experiments, and so to verify important theoretical results with any accuracy. There is another MHD laboratory available, however, and that is the Sun. The surface of the sun is a hot relatively dense plasma where the magnetic Reynolds number is very large, so the magnetic field is well and truly frozen into this fluid. However, we cannot change the experimental parameters. and we do not know what is going on below the level that we can see. so it is a less than perfect laboratory. Nevertheless, there are many interesting and varied phenomena that show the influence of MHD very well.

The visible surface of the sun is the photosphere, a fuzzy surface that we can see a little ways into, as into a cloud. It is composed of 90% hydrogen. 10% helium. with all the other elements as traces. Here energy rising from below encounters a steep declining temperature gradient as the surface layers cool by radiation into space. Convective instability produces active convection, with cells ("granules") averaging 700 miles in diameter, hot plasma rising in the center and sinking at the periphery after cooling a little. We observe a temperature of about 6000K looking straight down, closer to 5000K at the limb where we look obliquely and not so deep. The plasma emits a black-body spectrum. acting as an almost perfect emitter. The plasma cools rapidly above this level. becoming less dense and more transparent. In its lower temperature of about 4500K. neutral atoms absorb their characteristic lines, creating the Fraunhofer lines in the solar spectrum. This region is called the reversing layer, and extends some hundreds of miles vertically [21].

In solar eclipses, the moon covers the photosphere and the lower part of the reversing layer, but the upper part of this region is seen in its bright red Ha radiation at 656.3 nm. which dominates the flash spectrum observed at this instant, when all the solar Fraunhofer lines become bright emission lines. For this reason, this region is called the chromosphere, the lower part of the solar atmosphere. Above the reversing layer. the chromosphere becomes rarer and hotter, merging with the whitish corona above 13.000 miles. This is a rare plasma, in which the particles have very high kinetic energies. characteristic of $10^6$K, but there is no thermal equilibrium here, so, temperature is a doubtful concept. The magnetic Reynolds number is high throughout the solar atmosphere, and matter clings to the lines of force, sliding along them freely.

The Sun is almost perfectly spherical, subtending an angle of about 32' at mean distance. The equator is inclined to the ecliptic. and we see the north pole in September. the south pole in March, from our position 1.495977 x 101$^{3}$ cm distant. At the top of the atmosphere, we receive 1.37 x 10$^{6}$ erg/s-cm$^{2}$, which corresponds to a total luminosity of 3.8 x 103$^{3}$ erg/so The average density of the sun is 1.4 g/cm2, about the same as Jupiter's. Its total mass is 2 x 103$^{3}$ g, which makes its surface gravity about 28 times stronger than earth's. It is curious that its equatorial period of rotation is 25 days (27 days. as seen from earth that is revolving about the sun), but it rotates more and more slowly towards the poles. At a latitude of 45°, the rotational period is 28 days, and has been said to approach 33 days at the poles.

This difference in rotational periods is noted without much comment in astronomy texts. but it is really an extraordinary thing. In an ordinary liquid body. viscous forces would transfer angular momentum to the polar regions until they rotated at the same speed. In the sun, this momentum transfer also must occur, but the polar regions do not speed up. This means that some mechanism transfers angular momentum back to the equatorial regions at the same rate, and this mechanism can only be some magnetohydrodynamic effect, perhaps only in the outer layers of the sun. One side effect of this process may even be the appearance of sunspots when the necessary magnetic fields reach the sun's surface.

Sunspots, indeed, are evidence of solar magnetic fields. Magnetic flux enters or leaves the sun vertically at a sunspot, and the field magnitude can be thousands of gauss. We have already mentioned how this hinders convection in the region of high field. making the area cooler and darker than surrounding areas. The magnetic field of sunspots was discovered by Hale in 1908, and has been an interesting field of speculation since then. Sunspots occur in pairs of opposite polarity at roughly the same latitudes. so it seems that the field pierces the surface, bends over, and descends again in the companion spot. Polarities of the leading and following spots are reversed in the opposite hemisphere. and the polarities reverse in each successive 11.2-year sunspot cycle. The cycle is actually 22.4 years long. with two maxima in each cycle. No explanation is I-.:no\,\"n for this c\ck. but it is almost certainly magnetohydrodynamic. Sunspots do not

occur in polar or in equatiorial regions, but are restricted to mid-latitudes. Each maximum begins at high latitudes, and approaches the equator as the cycle progresses. The bigger the sunspot. the longer it lives. Large sunspots can live for weeks, and appear to cross the sun's disc more than once, but most do not. An average sunspot is 1000 miles in diameter miles in diameter, while a really large one could hold the earth.

There is a number of phenomena surrounding sunspots and associated with the chromosphere and corona that make magnetic fields manifest. There are spicules and faculae and flocculae, coronal streamers and rays, all apparently supported by magnetic fields. The most impressive and long-lived are the filaments or prominences. best seen in elevation at the limb of the sun in their reddish Ha light, great arches of luminous matter supported by the magnetic field. The field itself seems to be the longest-lived of all the spot phenomena. The form of the coronal streamers suggested a dipole field. but the sun has no strong general field, which cannot be stronger than a gauss or two. Originally. the sun was suspected of having a general field as large as 25 gauss. but that has pro\ed erroneous.

Solar flares are probably the result of some process of MHD acceleration of plasma in the active regions that breed sunspots. They are very hot. beginning with a blast of ultraviolet radiation that reaches the earth in a few minutes, creating extra ionization in the upper atmosphere. Then a jet of plasma is ejected that reaches the earth in about a day. causing ionospheric currents and ionization with magnetic storms. strong earth currents. disturbances to radio communication and the aurora borealis.

## CHARACTERISTIC METHOD

Many physical problems lead to the formulation of a quasi-linear system of first order equations. Such equations are linear in first derivatives of the dependent variables, but the coefficient may be functions of the dependent variable when these equations describe wave motion, a good understanding of many of the issues can be developed from the study of plane waves. Accordingly, we start with the case of two independent variables. The two variables often the time and space variable so we denote them by t and

x, and use corresponding terminology, but the discussion applies to any variable system if the dependent variables are $U_i$ (x, t), i = 1, 2,...n the general quasi-linear first order system is

$$A_{ij}\frac{\partial u_j}{\partial t}+a_{ij}\frac{\partial u_j}{\partial x}+b_i=0, \tag{1.27}$$

where the matrices A, a and vector b may be functions of $v_j$ -- $v_n$ as well as x and t.

In general, any one of the equations has different combination of $\partial u_i/\partial t$ and $\partial u_j/\partial x$ for $u_j$. That is it couples information about the rate of change of the different $u_j$ in different directions. One can not deduce information about the increments of all the $u_j$ for a step in any single direction but we are at liberty to manipulate the n equal to see whether this information can be obtained from some combination of them we therefore consider the linear combination.

$$l_i\left(A_{ij}\frac{u_j}{t}+a_{ij}\frac{u_j}{x}\right)+l_i\,b_i=0, \tag{1.28}$$

where the vector $l$ is a function of x,t,u and investigate whether $l$ can be chosen so that (1.28) takes the form

$$m_j\left(\beta\frac{\partial u_j}{\partial t}+\frac{\partial u_j}{\partial x}\right)+l\,b_i=0, \tag{1.29}$$

If this is possible (1.29) provides a relation between the directional derivative of all the $u_j$ in the single direction $(\alpha,\beta)$, when this is the case, it will be valuable to introduce curves in the (x, t) plane defined by the vector field $(\alpha,\beta)$. If x= x(n), t = T $(\eta)$ in the parametric representation, of a typical member of this family, the total derivation of $u_j$ on the curve is

$$\frac{du_j}{dn}=T'\frac{\partial u_j}{\partial t}+x'\frac{\partial u_j}{\partial_x},$$

without loss of generality, we may take

$\alpha = x'(\cap), \beta = T'(\cap)$

and write (1.29) as

$$m_j \frac{du_j}{dn} + l_j b_j = 0, \qquad (1.30)$$

The condition for (1.28) to be in the form (1.30) are

$l_i A_{ij} = m_j T', l_i a_{ij} = m_j x$

and we may eliminate the $m_j$ to give

$$l_i \left(A_{ij} x' - a^1_{ij} T'\right) = 0, \qquad (1.31)$$

There are equation for the multiplers $I_i$ and the direction (x', T'), since they are homogeneous in the $l_i$, a necessary and sufficient condition for a nontrivial solution is that the determinate.

$$\left(A_{ij} x' - a_{ij} T'\right) = 0, \qquad (1.32)$$

This is a condition on the direction of the curve such a curve in said to be a characteristic and the corresponding equation (1.30) in said to be characteristic form.

## WITHAM'S RULE

Chester has studied the motion of a shock wave down a non uniform tube on the basis of a linearlized theory in which the changes in the tube area and the consequent changes in shock strength have been assumed to be small. In this linearlized theory, the solutions breaks down when the flow behind the shock wave is nearly sonic Chisnel and Witham [7] have shown that Chesters work could be simplified and extended one minor simplification is that where as the Chester worked with the full three dimensional equations and performed and averaging process in the course of his analysis. It is sufficient to work from the outset with the one dimensional formulation. Witham has shown that the motion of the shock can be found in a simple way without solving the

equations for the flow behind the shock in detail. This method can be given as the following rule.

The relevant equations of motions are first written in characteristic form. Then the rule is to apply the differential relation which must be satisfied by the flow quantities along a characteristic to the flow quantities just behind the shock wave. Together with the shock relations which express these values in terms of the shock strength, the rule determines the change in the shock strength. In fact it is found to the accurate even when the total changes in the shock strength are not small although a full understanding of this fact is still lacking.

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# Chapter - II
# *Propagation of Exponential Magnetoradiative Shock Waves*

## INTRODUCTION

In this present chapter the self similar model of exponential spherical, cylindrical and plane shock waves is studied, taking magnetic radiative heat flux into account where total energy of the wave is variable and atmosphere is uniform. Similarty method for shock waves have studied wang [1], Koch [2], Helliwell [3], Ray and Banerjee [4] and other have investigated the propagation of plane shock waves in optically thick and thin limit cases of gas. In detail Gusev [5] and Ranga Rao and Ramana [6] have studied the problem of unsteady self similar motion of a gas displaced by a piston according to an exponential law. Verma and Singh [7] and Singh & Srivastava [8] have considered the problems of spherical shock waves in an exponentially increasing medium under the law of uniform pressure. Srivastava [9] has studied the problem of magnetoradiative shock in a conducting plasma.

In this chapter we discussed the strong shock waves in a exponential spherical, cylindrical and plane shock waves in a uniform atmosphere with magnetic radiative effects. The similarity solution have been developed when radiation heat is more important than the radiation pressure & radiation energy and opaque the shock to be transparent and isothermal. The total energy of waves as cube of shock radius.

The shock simulation is time dependent, and we assume the density to increase according power law. A solar flare accompanied by enhanced coronal temperature possibly exceeding $4x16^6$K gives rise to an ejection of plasma at a velocity of 500-1500 kms$^{-1}$ in interplanetary space, and to a shock wave propagating outwards from the sun and reaching orbit of the Earth. In this case the shock wave will have a region where density increases in the direction of propagation J.B. Singh and P.R. Vishwakarma [10] have obtain and extent exponential shock with radiative heat flux. V.K. Singh [11] has find solution in conducting medium, R.F. Chisnell [12] has obtain and extent an analysis descriptions of concerning shock waves. Dheeraj Bhardwaj [13] has obtain and formation

of shock waves in magneto gas dynamics flows. Self similar flow behind a shock wave in a gravitating or non gravitating gas with heat conduction and radiation heat flux, were obtained by J.P. Vishwarkarma & Arvind K.C. [14]. The shock wave propagate in an uniform atmosphere which is assumed to be at rest. The similarity solution have been developed when the radiation pressure and radiation energy is present viscosity and the solar gravity have been neglected.

## EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The equation of flow behind a spherical, cylindrical and plane shock waves where $j = 0,1,2$ corresponding to plane, cylindrical & spherical and $\upsilon = 0$ for plane and $\upsilon = 1$ for cylindrical & spherical both.

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial r} + \frac{j\rho u}{r} = 0 \qquad (2.01)$$

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial t} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\upsilon h^2}{\rho r} = 0 \qquad (2.02)$$

$$\frac{dh}{dt} + \frac{h\,\partial u}{\partial r} + \frac{\upsilon h u}{r} = 0 \qquad (2.03)$$

$$\frac{dE}{dt} + P\frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r^j}\frac{\partial}{\partial R}\left(Fr^j\right) = 0 \qquad (2.04)$$

$$E = \frac{P}{(\gamma-1)\rho} \qquad (2.05)$$

$$P = \Gamma P T \qquad (2.06)$$

Where u, $\rho$, P, and T are the velocity, density, pressure and temperature respectively $\Gamma$ being the gas constant are $\gamma$ the ratio of specific heat while F denotes the heat flux.

Assuming local thermodynamic equilibrium and taking Rosseland's diffusion approximation, we have

$$F = -\frac{c\mu}{3}\frac{\partial}{\partial r}\left(\sigma T^4\right) \qquad (2.07)$$

Where $\frac{\sigma c}{4}$ is the stefan Boltmann constant. C the velocity of light, $\mu$ the mean free path of radiation is the function of density and temperature following wang [1] we take

$$\mu = \mu_0 \, \rho^{\alpha} \, T^{\beta} \tag{2.08}$$

$\mu_0$, $\alpha$, &$\beta$ are being constants

The surface of constant discontinuity moves with time according to an exponential law.

$$\bar{r} = A \ \exp(mt) \ m > 0 \tag{2.09}$$

and since we have the flow is self similar, the shock will also moves with time according to an exponential law

$$R = B \ \exp(mt) \tag{2.10}$$

where $\bar{r}$ is the radius of inner expanding surface, R is the shock radius A, B and m are dimensional constant. The disturbance is headed by an isothermal shock therefore. The boundary conditions are

$$u_1 = \left[1 - \frac{1}{\gamma M^2}\right] v \tag{2.11}$$

$$\rho_1 = \gamma M^2 \ \rho_0 \tag{2.11}$$

$$p_1 = \rho_0 v^2 \tag{2.12}$$

$$F_1 = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \rho_0 v^3 \tag{2.13}$$

$$h_1 = \gamma \, M^2 \, h_0 \tag{2.14}$$

where subscript 0 and 1 denote the regions immediately ahead and behind shock front respectively and v is the shock velocity M denotes mach number.

## SIMILARITY SOLUTIONS

The similarity transformation for the problems under consideration are

$$\eta = \frac{r}{B \exp(mt)} \tag{2.15}$$

$$u = vV(\eta) \tag{2.16}$$

$$\rho = \rho_0 \, G(\eta) \tag{2.17}$$

$$P = \rho_0 \, v \, P(\eta) \tag{2.18}$$

$$F = \rho_0 \, v^3 \, Q(\eta) \tag{2.19}$$

$$h = \sqrt{\rho_0} \, v \, H(\eta) \tag{2.20}$$

the variable $\eta$ assumes the value 1 at the shock and $\eta$ on the inner expanding surface. This enables us to express the radius of inner expanding surface.

$$\bar{r} = \eta R \tag{2.21}$$

at $\eta=1$ & $r = R$ from eq. (2.15)

$R = B \exp(mt)$ by (2.10)

$$\frac{dR}{dt} = m B \exp(mt)$$

or $$\frac{dR}{dt} = mBe^{mt}$$

or shock velocity

$$v = mR \tag{2.22}$$

$$\frac{v}{R} = m$$

By equation (2.07)

$$F = -\frac{c\mu}{3} \frac{\partial}{\partial r}\left(\sigma T^4\right)$$

Eq. (2.06 & 2.08)

$$F = \frac{-c\sigma\mu_0}{3\Gamma^{u+\beta}} \rho^{\alpha-\beta} P^{\beta} \frac{\partial}{\partial r}\left(\frac{P^4}{\rho^4}\right)$$

using (2.17), (2.18), (2.19)

$$\rho_0 v^3 Q(\eta) = \frac{-c\sigma\mu_0}{3\Gamma^{u+\beta}} G^{\alpha-\beta-u} P^{(\beta+4)} \left[\frac{P'}{P} - \frac{G'}{G}\right]$$

Using (2.21) & (2.22)

$$Q = -\frac{4mc\,\mu_0\,\sigma\rho_0^{(\alpha-1)}}{3\Gamma^{\beta+4}}\,P^{\beta+4}\left[\frac{P'}{P} - \frac{G'}{G}\right] \tag{2.23}$$

with $\beta = -2, \alpha$ remaining arbitary ($0 < \alpha < 2$) and

$N = \dfrac{4mc\mu_0\,\sigma\rho_0^{(\alpha-1)}}{3\Gamma^{(\beta+4)}}$ = a dimensionless radiation parameter.

The equation (2.15) to (2.20) are used to reduce the equation (2.01) to (2.06) and (2.23) in new form by equation (2.23), we get

$$Q = -\,NG^{\alpha-\beta-4}\,P^{\beta+4}\left[\frac{P'}{P} - \frac{G'}{G}\right] \quad \text{since} \quad \beta = -2$$

$$Q = -\,NG^{(\alpha-1)-1}\,P^2\left[\frac{P'}{P} - \frac{G'}{G}\right]$$

$$V' = \frac{\dfrac{QG^{(1-\alpha)}(\eta - V)}{NP} - jPV + (\eta - V)H^2\,\upsilon V - \upsilon H^2 - G}{\left[G(\eta - V)^2 - P - \eta H^2(1 - V)(\eta - V)\right]} \tag{2.24}$$

By equation (2.17)

$$\rho = \rho_0\,G(\eta)$$

$$\frac{\partial\rho}{\partial t} = \rho_0\,G'(\eta)\frac{\partial\eta}{\partial t}$$

$$\because \eta = \frac{r}{B\exp(mt)}$$

$$\frac{\partial\eta}{\partial t} = \frac{-rm}{B\exp(mt)}$$

as $\quad \dfrac{\partial\eta}{\partial t} = -\,\eta m$

$$\frac{\partial\rho}{\partial t} = -\rho_0\,G'\,\eta m$$

also $\quad \rho = \rho_0\,G(\eta)$

$$\frac{\partial\rho}{\partial r} = \rho_0\,G'\,\frac{\partial\eta}{\partial r}$$

$\because \quad \eta = \frac{r}{B} e^{-mt}$

$$\frac{\partial \eta}{\partial r} = \frac{1}{B} e^{-mt}$$

$$\frac{\partial \eta}{\partial r} = \frac{\eta}{r}$$

as $\quad \frac{\partial \rho}{\partial r} = \rho_0 G' \frac{\eta}{r}$

also $\quad u = vV(\eta)$

$$\frac{\partial u}{\partial r} = vV'(\eta) \frac{\partial \eta}{\partial r}$$

as $\quad \frac{\partial u}{\partial r} = vV' \frac{\eta}{r}$

from equation (2.01)

$$\frac{d\rho}{dt} + \rho \frac{\partial u}{\partial r} + j \frac{\rho u}{r} = 0$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + j \frac{\rho u}{r} = 0 \qquad \text{(a)}$$

substituting these values $\frac{\partial \rho}{\partial t}, \frac{\partial \rho}{\partial r}, \frac{\partial u}{\partial r}, \rho \,\&\, u$ in Eq. (a)

$$-\rho_0 G' \eta m + \upsilon v \rho_0 G' \frac{\eta}{r} + \rho_0 G \upsilon v' \frac{\eta}{r} + \frac{j\rho_0 G \upsilon v}{r} = 0$$

$$G' = \frac{G(\eta V' + jV)}{\eta(\eta - V)} \qquad \left(m = \frac{v}{R}\right) \qquad (r = R) \qquad (2.25)$$

Equation (2.02)

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\upsilon h^2}{\rho r} = 0$$

$$\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\upsilon h^2}{\rho r} = 0 \qquad \text{(b)}$$

By equation (2.16)

$u = vV(\eta)$

$$\frac{\partial u}{\partial t} = vV'(\eta)\frac{\partial \eta}{\partial t} + V(\eta)\frac{\partial v}{\partial t}$$

since $\frac{\partial v}{\partial t} = m^2R$ & $\frac{\partial \eta}{\partial t} = -\eta m$

$$\frac{\partial u}{\partial t} = -mv\eta V'(\eta) + mv\,V(\eta)$$

& $\quad \frac{\partial u}{\partial r} = \frac{vV'(\eta)}{r}\frac{\eta}{r}$

$$P = \rho_0 v\,P(\eta)$$

$$\frac{\partial P}{\partial r} = \rho_0 v\,P^1\frac{\partial \eta}{\partial r} = \rho_0\frac{vP'\eta}{r}$$

By equation (2.20)

$$h = \sqrt{\rho_0}\;v\,H(\eta)$$

$$\frac{\partial h}{\partial r} = \sqrt{\rho_0}\;v\,H'\frac{\partial \eta}{\partial r} = \sqrt{\rho_0}\;v\,H'\frac{\eta}{r}$$

Substituting these values equation (b)

$$-mv\eta V' + mvV + mv\,VV' + mv\frac{P^1}{G} + \frac{\sqrt{\rho_0}\,vH}{\rho_0 G}\sqrt{\rho_0}\;mH'\eta + \frac{\upsilon\rho_0 v^2H^2}{\rho_0 Gr} = 0$$

$$-\eta V' + V + VV' + \frac{P'}{G} + \frac{HH'\eta}{G} + \frac{\upsilon H^2}{G} = 0$$

$$P^1 = \left[HH^1\eta - \upsilon H^2\right] - G\left[V'(\eta - V) - V\right] \qquad (2.26)$$

Equation (2.03)

$$\frac{dh}{dt} + \frac{h\,\partial u}{\partial r} + \frac{\upsilon\,hu}{r} = 0$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \upsilon\frac{hu}{r} = 0 \qquad (c)$$

By equation (2.20)

$$h = \sqrt{\rho_0}\;vH(\eta)$$

$$\frac{\partial h}{\partial t} = \sqrt{\rho_0}\left[H'\frac{\partial \eta}{\partial t}.v + H(\eta)\frac{\partial v}{\partial t}\right]$$

$$= \sqrt{\rho_0}\left[H^1 v\left(-\eta m\right) + Hmv\right]$$

$$\frac{\partial h}{\partial t} = \sqrt{\rho_0}\ vm\left[-H'\eta + H\right]$$

By equation (2.16)

$$u = vV(\eta)$$

$$\frac{\partial u}{\partial r} = vV'\frac{\partial \eta}{\partial r}$$

$$\frac{\partial u}{\partial r} = vV'\frac{\eta}{r}$$

By equation (2.20)

$$h = \sqrt{\rho_0}\ vH(\eta)$$

$$\frac{\partial h}{\partial r} = \sqrt{\rho_0}\ vH'\frac{\partial \eta}{\partial r}$$

$$\frac{\partial h}{\partial r} = \sqrt{\rho_0}\ vH'\frac{\eta}{r}$$

Substituting these values in equation (c)

$$\sqrt{\rho_0}\ vm\left[-H'\eta + H\right] + vV\sqrt{\rho_0}\ vH'\frac{\eta}{r} + \sqrt{\rho_0}\ vH.\,vV'\frac{\eta}{r} + \upsilon\frac{\sqrt{\rho_0}\ vHvV}{r} = 0$$

$$\sqrt{\rho_0}\ v.\frac{v}{r}\left[-H'\eta + H\right] + vv\frac{\eta}{r}\sqrt{\rho_0}\ vH' + \sqrt{\rho_0}\ v.v.V'H\frac{\eta}{r} + \sqrt{\rho_0}\ \frac{v.v}{r}\upsilon HV = 0$$

$$\left[-H'\eta + H\right] + VH' + V'H\eta + \upsilon HV = 0$$

$$H' = (1 - V)\left[HV' + \frac{\upsilon VH}{\eta}\right] \qquad (2.27)$$

By equation (2.04)

$$\frac{dE}{dt} + P\frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r^j}\frac{\partial}{\partial r}\left(Fr^j\right) = 0$$

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} + P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) + Pu\frac{\partial}{\partial r}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r^j}\frac{\partial}{\partial r}\left(Fr^j\right) = 0$$

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} - \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} - \frac{uP}{\rho^2}\frac{\partial \rho}{\partial r} + \frac{1}{\rho}\frac{\partial F}{\partial r} + \frac{jF}{r\rho} = 0$$

$$E = \frac{P}{(\gamma - 1)\rho} \qquad \text{(d)}$$

$$\frac{\partial E}{\partial t} = \frac{1}{(\gamma - 1)}\left[\frac{1}{\rho}\frac{\partial P}{\partial t} - \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}\right]$$

&

$$\frac{\partial E}{\partial r} = \frac{1}{(\gamma - 1)}\left[\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{P}{\rho^2}\frac{\partial \rho}{\partial r}\right]$$

By equation (2.17)

$$\rho = \rho_0 \, G(\eta)$$

$$\frac{\partial \rho}{\partial r} = \rho_0 \, G' \frac{\partial \eta}{\partial r}$$

$$\frac{\partial \rho}{\partial r} = \rho_0 \, G' \frac{\eta}{r}$$

&

$$\frac{\partial \rho}{\partial t} = \rho_0 \, G' \frac{\partial \eta}{\partial t}$$

$$\frac{\partial \rho}{\partial t} = \rho_0 \, G' \, \eta m$$

By equation (2.19)

$$F = \rho_0 \, v^3 \, Q(\eta)$$

$$\frac{\partial F}{\partial r} = \rho_0 \, v^3 \, Q' \frac{\partial \eta}{\partial r}$$

$$\frac{\partial F}{\partial r} = \rho_0 \, v^3 \, Q' \frac{\eta}{r}$$

By equation (2.18)

$$P = \rho_0 \, v \, P(\eta) \qquad \text{(d)}$$

$$\frac{\partial P}{\partial t} = \rho_0 \left[\frac{\partial v}{\partial t} P + v P' \frac{\partial \eta}{\partial t}\right]$$

$$= \rho_0 \left[m v P + v P'(-\eta m)\right]$$

$$\frac{\partial P}{\partial t} = \rho_0 \ mv \left[P - P'\eta\right]$$

again by equation (2.18)

$$\frac{\partial P}{\partial r} = \rho_0 \, v P' \, \frac{\eta}{r}$$

Substituting these values in equation (d), we get

$$\frac{1}{(\gamma-1)}\left[\frac{1}{\rho_0 G}\ \rho_0 \, mv\left(P - P'\eta\right) - \frac{\rho_0 \, vP.}{\rho_0^2 G^2}\ \rho_0 \, G' \, \frac{\eta}{r}\right]$$

$$+\frac{vV}{(\gamma-1)}\left[\frac{\rho_0 \, vP'}{\rho_0 G}\ \frac{\eta}{r} - \frac{\rho_0 vP}{\rho_0^2 G^2}\ \rho_0 \, G' \, \frac{\eta}{r}\right] - \frac{\rho_0 \, vP}{G^2 \rho_0^2}\left(-\rho_0 \, G' \, m\,\eta\right) - \frac{v\,V.\rho_0 \, P\,v.\rho_0 \, G'}{\rho_0^2 \, G^2}\ \frac{\eta}{r}$$

$$+\frac{1}{\rho_0 G}\ \rho_0 \, v^3 \, Q' \, \frac{\eta}{r} + j\,\frac{\rho_0 \, v^3 \, Q}{r.\rho_0 \, G} = 0$$

$$= \frac{1}{(\gamma-1)}\left[\frac{(P-P'\eta)}{G} - \frac{PG'\eta}{G^2}\right] + \frac{V}{(\gamma-1)}\left[\frac{P'\eta}{G} - \frac{PG'\eta}{G^2}\right] + \frac{PG'\eta}{G^2} - \frac{VPG'\eta}{G^2} + \frac{Q'\eta}{G} + \frac{jQ}{G} = 0$$

$$Q' = \frac{P}{(\gamma-1)} + \frac{PG;}{G\eta}\ \frac{(2+V-\gamma)}{(\gamma-1)} - \frac{Qj}{\eta} - \frac{VP'}{\eta(\gamma-1)} \qquad (2.28)$$

The shock conditions (2.11)-(2.14) are transformed into the following forms (at $\eta = \eta_0$)

$$u_1 = \left[1 - \frac{1}{\gamma M^2}\right] v$$

$$v\,V(\eta) = \left[1 - \frac{1}{\gamma M^2}\right] v$$

at $\eta = 1$ & $r = R$

$$V\,(1) = \left[1 - \frac{1}{\gamma M^2}\right] \qquad (2.29)$$

By condition (2.12)

$$\rho_1 = \rho_0 \ \gamma M^2$$

$$\rho_0 \, G(\eta) = \gamma M^2 \, \rho_0$$

at $\eta = 1$ & $r = R$

$$G\,(1) = \gamma M^2 \qquad (2.30)$$

By condition (2.13)

$$P_1 = \rho_0 v^2$$

$$\rho_0 v^2 P(\eta) = \rho_0 v^2$$

at $\eta = 1$

$$P(1) = 1 \qquad (2.31)$$

Condition (2.14)

$$F_1 = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right]\rho_0 v^3$$

$$\rho_0 v^3 Q(\eta) = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right]\rho_0 v^3$$

at $\eta = 1$

$$Q(1) = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \qquad (2.32)$$

By condition (2.15)

$$h_1 = \gamma M^2 h_0$$

$$\sqrt{\rho_0}\, v H(\eta) = \gamma M^2 \frac{\sqrt{\rho_0}}{M} . v$$

at $\eta = 1$

$$H(1) = \gamma M \qquad (2.33)$$

Thus the transformed equations of motion are

$$V' = \frac{\dfrac{QG^{(1-\alpha)}(\eta - V)}{NP} - j PV + (\eta - V) H^2 - \upsilon V - \upsilon H^2 - G}{\left|G(\eta - V)^2 - P - \eta H^2 (1 - V)(\eta - V)\right|} \qquad (2.24)$$

$$G' = \frac{G(\eta V' + jV)}{\eta(\eta - V)} \qquad (2.25)$$

$$P' = \left|HH'\eta - \upsilon H^2\right| - G\left|V'(\eta - V) - V\right| \qquad (2.26)$$

$$H' = (1 - V)\left[HV' + \frac{\upsilon HV}{\eta}\right] \qquad (2.27)$$

$$Q' = \frac{P}{(\gamma-1)} + \frac{PG'}{G\eta}\frac{(2+V-\gamma)}{(\gamma-1)} - \frac{Qj}{\eta} - \frac{VP'}{\eta(\gamma-1)} \tag{2.28}$$

The appropriate transformed shock conditions are

$$V_{(1)} = \left[1 - \frac{1}{\gamma M^2}\right] \tag{2.29}$$

$$G_{(1)} = \gamma M^2 \tag{2.30}$$

$$P_{(1)} = 1 \tag{2.31}$$

$$Q_{(1)} = \frac{1}{2}\left[\frac{1}{\gamma M^2} - 1\right] \tag{2.32}$$

$$H_{(1)} = \gamma M \tag{2.33}$$

Where primes denotes differentiation with respect $\eta$.

## NUMERICAL SOLUTION AND RESULT

For exhibiting the numerical solutions, it is convenient to write the flow variables in the non dimensional forms as

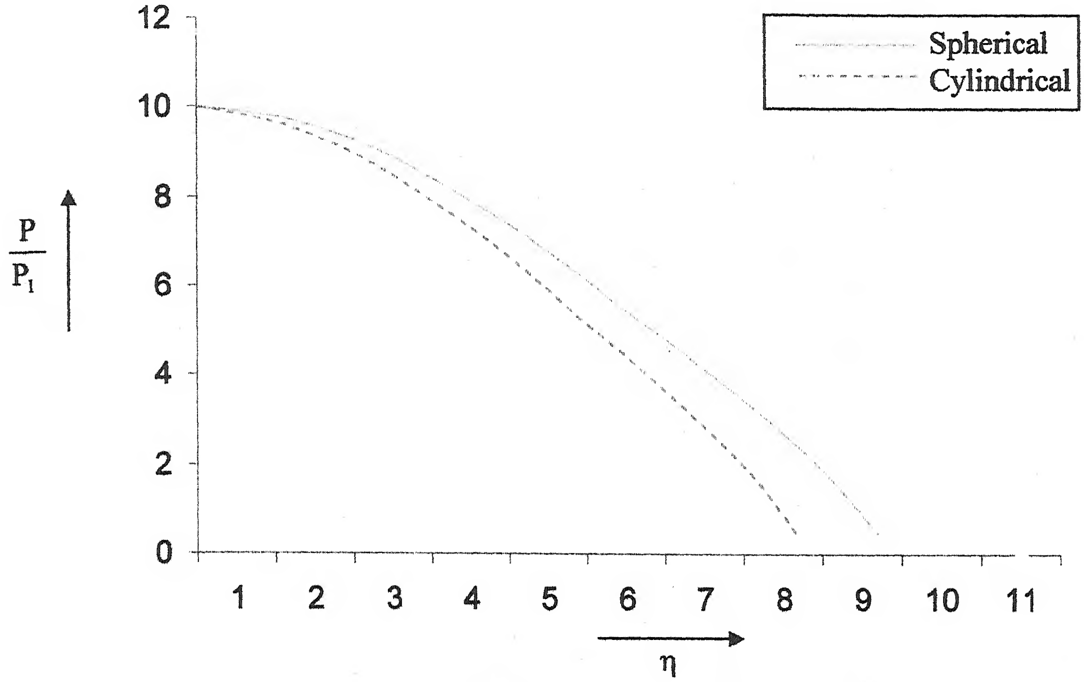
$$\frac{u}{u_1} = \frac{V}{V_{(1)}}, \qquad \frac{\rho}{\rho_1} = \frac{G}{G_{(1)}}$$

$$\frac{p}{p_1} = \frac{P}{P_{(1)}}, \qquad \frac{F}{F_1} = \frac{Q}{Q_{(1)}}$$

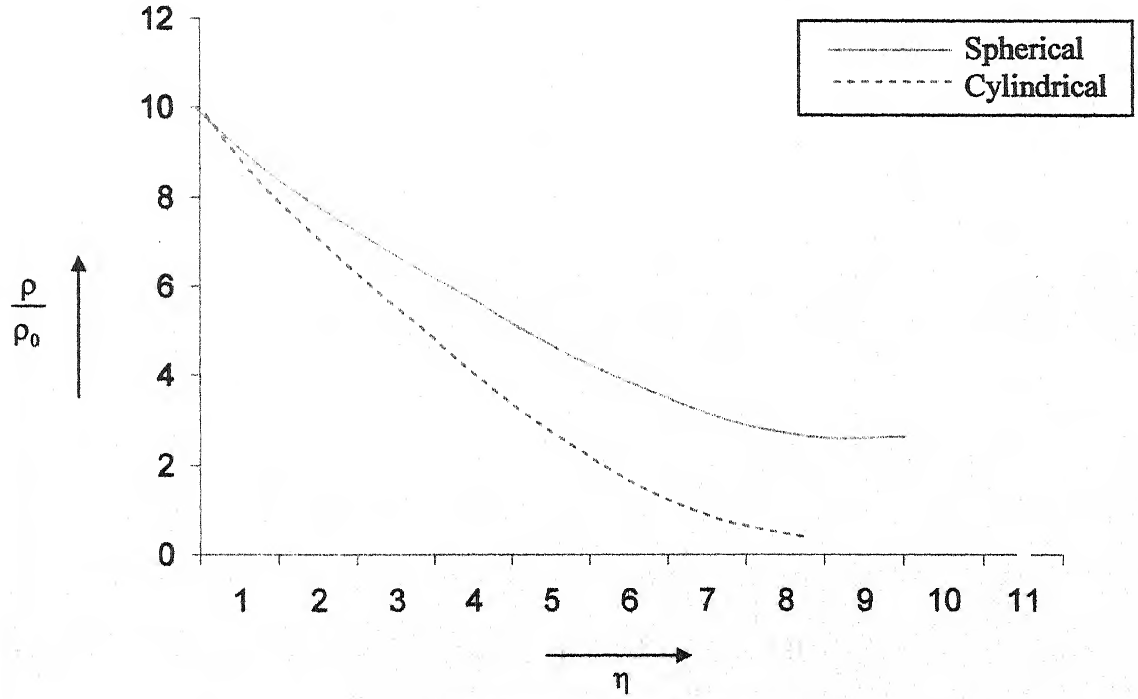
$$\frac{h}{h_1} = \frac{H}{H_{(1)}}$$

The numerical integration was carried out through using software matlab, for certain choice of parameter and reproduced in graphical form and nature of field variables is illustrated through them. We have calculated our result for following data.

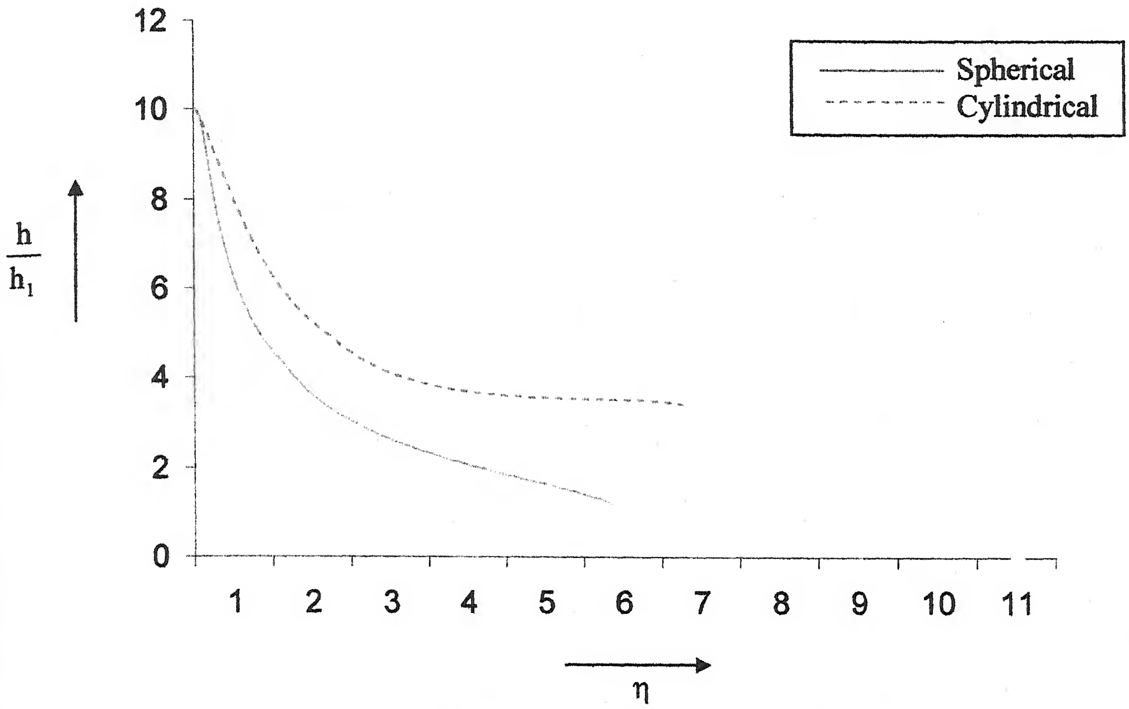
$\gamma = 1.4 \qquad M^2 = 1.01, \qquad \alpha = 0.25$



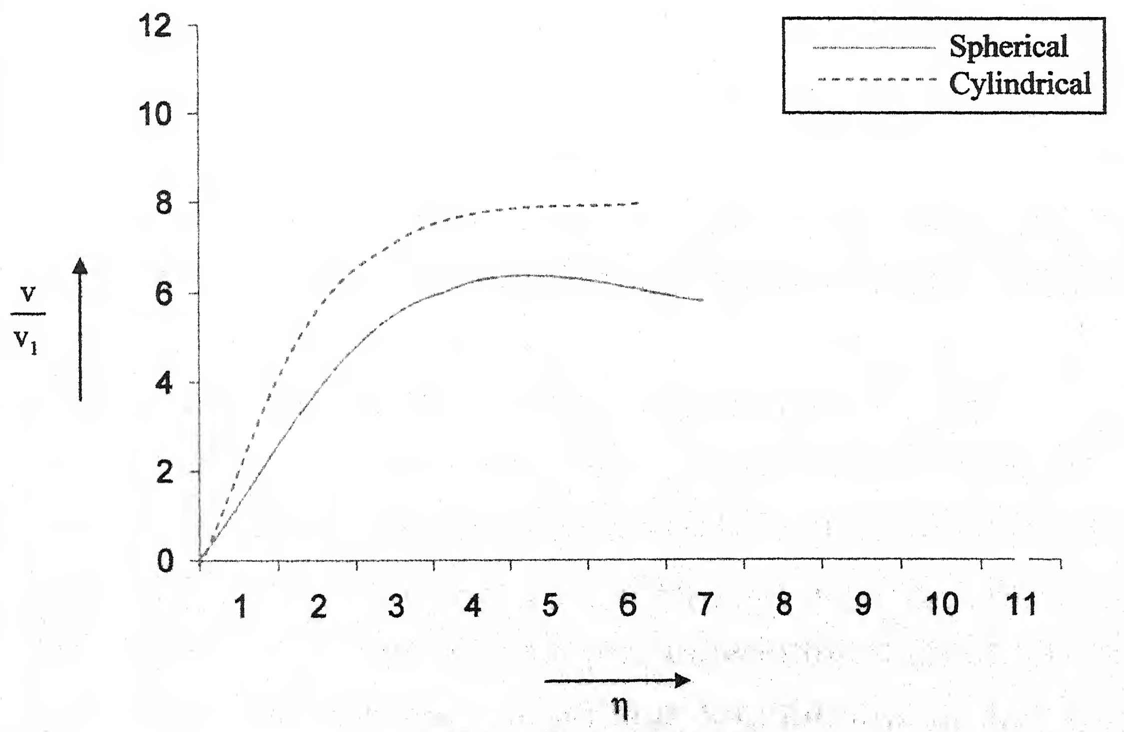

(Pressure Distribution)




(Density Distribution)




(Magnetic Distribution)




(Velocity Distribution)

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# Chapter - III

# *Self Similar Power Driven Isothermal Flow Behind Cylinderical Shock in Monochromatic Radiation with Gravitational force*

## INTRODUCTION

A self similar theoretical model of power driven isothermal expansion in a non homogeneous medium in the presence of monochromatic radiation with gravitational force is considered. The result discussed depends upon variation of the flow variables behind the shock which are displayed graphically. Gas is assume to be gray and opaque shock.

The theory of self similar flow behind shock waves in a non homogeneous medium has been discussed by many authors Sakurai [1], cole [2], Kopal [3] etc. Singh [4] has discussed the problem of propagation of shock waves in a one dimensional plane model of a non-homogeneous medium. Also Verma & Vishwakarma [5] have considered the problem of propagation of plane Cylindrical and spherical magnetogasdynamics, shock waves in a decreasing density medium using Whithams [6] rule. Sedov [7] made it possible to analyze certain classes of self similar solutions in a number of problems with disturbed energy release with the help of method of theory of dimensionally Onkar Nath [8] presented a theoretical model of cylindrical magnetogasdynamifccs shock waves under the action of monochromatic radiation in non-uniform steller atmosphere following work of Khudryautsev [9] on self similar problem of a motion of a gas under the action of monochromatic radiation. Nath [10], Manoj Pandey and V.D. Sharma [11] have discussed similarity analysis and find exist solution magnetogasdynamic equation further studied a model of cylindrical shock waves in a non-uniform rotating atmosphere under the action of monochromatic radiation, where he omitted magnetic field effect.

We have studied the propagation of cylindrical shock wave in a magnetogasdynamics rotating non uniform atmosphere in the presence of monochromatic radiation and gravitation. The shock is assumed to be propagating in a conducting

medium at rest with density varing as $r^{\beta}$ $(-2<\beta\leq 0)$. The magnetic field distribution varies as $r^{\alpha}$ $(\alpha<0)$ and is directed tangential to the advancing shock front. The radiation flux moves through the gas with a constant intensity in the direction opposite to that of the propagation of shock wave. Further, the rotating gas does not radiate itself and energy in absorbed only behind the shock wave. The radiation pressure and energy are very less hence neglected.

For isothermal expansion of the plasma the internal heat generation per unit mass is identical for all elements of the fluid behind surface of discontinuity and is only a function of time so that for isothermal case.

$$v=\mu=\sigma=0 \tag{i}$$

The medium in which expansion takes place is heterogeneous. The shock radius is a function of time. Thus density and shock radius have been assumed to obey the following power law.

$$\rho_0 = A\,t^{\alpha} \tag{ii}$$

$$R = Bt^{\beta} \tag{iii}$$

$$\rho_0 = \rho\, R^{\beta} \qquad (-2<\beta\leq 0) \tag{iv}$$

Where $\rho_0$ is the density of the ambient medium and R is the shock radius. Here A, B, $\alpha$ and $\beta$ are constants.

## EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

In accordance with the above assumption the motion of an inviscid perfect gas in a magnetogasdynamics rotating non-uniform medium in presence of monochromatic radiation and gravitation can be described by the following system of differential equation

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial t} + \frac{\rho u}{r} = 0 \tag{3.01}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{hu}{r} = 0 \tag{3.02}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{h^2}{\rho r} + \frac{Gm}{r} - \frac{v^2}{r} = 0 \tag{3.03}$$

$$\frac{d}{dt}(v.\,r)=0 \tag{3.04}$$

$$\frac{\partial m}{\partial r}=2\pi\rho r \tag{3.05}$$

$$\frac{\partial e}{\partial t}+u\frac{\partial e}{\partial r}-\frac{P}{\rho^2}\left[\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right]+\frac{1}{\rho r}\frac{\partial}{\partial r}(jr)=0 \tag{3.06}$$

$$\frac{\partial j}{\partial}=kj \tag{3.07}$$

Where u, P, ρ, h, m, v, j and e are radial component of velocity, pressure, density, magnetic field, mass per unit volume, azimuthal component of velocity monochromatic radiation flux, energy per unit mass at a radial distance r and time t respectively G represents the gravitational constant and K is the absorption coefficient.

$$e=\frac{P}{\rho(\gamma-1)} \tag{3.08}$$

Treating the cylindrical shock front heading the expanding plasma as a surface of discontinuity. The jump conditions may written Whitam [6] as

$$P_1=\frac{2}{(\gamma+1)}\rho_0\dot{R}^2 \tag{3.09}$$

$$\rho_1=\frac{\gamma+1}{(\gamma-1)}\rho_0 \tag{3.10}$$

$$h_1=\frac{\gamma+1}{(\gamma-1)}h_0 \tag{3.11}$$

$$u_1=\frac{2}{\gamma+1}\dot{R} \tag{3.12}$$

$$v_1=\frac{2}{\gamma+1}\dot{R} \tag{3.13}$$

$$e_1=\frac{2\dot{R}^2}{(\gamma+1)} \tag{3.14}$$

$$m_1=m_0=2\pi\frac{\rho^* r^{2+\beta}}{(2+\beta)} \tag{3.15}$$

Where suffixes '1' denotes state jump behind shock surface and '0' denotes state of flow variables jump ahead of the shock surface and the dot's stands for differentiation with respect to time.

The Alfven mach number and usual mach number are defined as

$M_A^2 = \dfrac{\rho_0 \dot{R}^2}{h_0^2}$ *and* $M^2 = \dfrac{\rho_0 \ \dot{R}^2}{\gamma P_0}$ respectively where $\dot{R} = \dfrac{dR}{dt}$ is the speed of shock.

The absorption co-efficient k, is considered as

$$k = k_0 \ \rho^n \ P^m \ j^q \ r^s \ t^l \tag{3.18}$$

where the dimension of constant $k_o$ is given by

$$[K_0] = M^{-n-m-q} \ L^{3n+m-s} \ T^{2m+3q-l} \tag{3.19}$$

moreover the dimensions less constants $J_0$, $P_0$, $\rho_0$ are related as

$$j_0 = P_0^{3/2} \ \rho_0^{-1/2} \tag{3.20}$$

under the equilibrium condition, we have from ( 3.03 )

$$G = -\left[\frac{1}{(\gamma M^2)} + \frac{1}{(2M_A^2)}\right] (2+\beta)\ (1+\beta)\left(\frac{dR}{dt}\right)^2 / \pi \rho^* \ r^{\beta+2} \tag{3.21}$$

## SIMILARITY SOLUTIONS

Let us consider the solution of the equation in the form

$$u = \dot{R} \ U(\eta) \tag{3.22}$$

$$\rho = \rho_0 \ g(\eta) \tag{3.23}$$

$$P = \rho_0 \ \dot{R}^2 \ X(\eta) \tag{3.24}$$

$$h = \sqrt{\rho_0} \ \ \dot{R} \ H(\eta) \tag{3.25}$$

$$v = \dot{R} \ V(\eta) \tag{3.26}$$

$$m = m_1 \ W(\eta) \tag{3.27}$$

$$J = J_0 \ J(\eta) \tag{3.28}$$

$$e = \dot{R}^2 \ E(\eta) \tag{3.29}$$

where $\eta$ is a non dimensional parameter defined to be

$$\eta = \frac{r}{R(t)} \tag{3.30}$$

and thus $\eta=1$ at shock front $r = R$

## SOLUTIONS OF EQUATIONS OF MOTION

Equations (3.01) to (3.07) may also be transformed with the help or non-dimensional variables given in (3.22) – (3.30)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \tag{3.01}$$

By equation (3.23)

$$\rho = \rho_0\, g(\eta)$$

$$\frac{\partial \rho}{\partial t} = \rho_0\, g'\frac{\partial \eta}{\partial t} + g\frac{\partial \rho_0}{\partial t}$$

$$\eta = \frac{r}{R}$$

$$\frac{\partial \eta}{\partial t} = \frac{-r}{R^2}\,\dot{R}$$

$$\frac{\partial \eta}{\partial t} = \frac{-\eta}{R}\,\dot{R}$$

By equation (ii)

$$\rho_0 = A\,t^{\alpha} \qquad R = Bt^{\beta}$$

$$\frac{\partial \rho_0}{\partial t} = \alpha A\, t^{\alpha-1} \qquad \left[\dot{R} = \frac{\beta R}{t}\right]$$

$$\frac{\partial \rho_0}{\partial t} = \frac{\alpha\,\rho_0}{t} \qquad \left[t = \frac{\beta R}{\dot{R}}\right]$$

$$\frac{\partial \rho}{\partial t} = \rho_0\, g'\left(\frac{-\eta}{R}\,\dot{R}\right) + g\,\frac{\alpha\,\rho_0}{t}$$

$$= \rho_0\, g'\,\frac{-\eta\dot{R}}{R} + \frac{g\,\alpha}{\beta}\,\frac{\dot{R}}{R}\,\rho_0$$

$$\frac{\partial \rho}{\partial t} = \frac{\rho_0}{R}\,\dot{R}\left[-g'\eta + g\,\frac{\alpha}{\beta}\right]$$

By equation (3.23)

$$\rho = \rho_0\, g(\eta)$$

$$\frac{\partial \rho}{\partial r} = \rho_0\, g'\, \frac{\partial \eta}{\partial r}$$

$$\frac{\partial \rho}{\partial r} = \frac{\rho_0\, g'}{R}$$

By condition (3.22)

$$u = \dot{R}\, U(\eta)$$

$$\frac{\partial u}{\partial r} = \dot{R}\, U' \frac{\partial \eta}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{\dot{R}\, U'}{R}$$

substituting these values in equation (3.01)

$$-\rho_0\, g'\, \eta\, \frac{\dot{R}}{R} + \frac{\alpha}{\beta}\, g\, \rho_0\, \frac{\dot{R}}{R} + \dot{R}\, U\, \frac{\rho_0\, g'}{R} + \frac{\rho_0\, g\, \dot{R}\, U'}{R} + \rho_0\, g\, \frac{\dot{R}U}{\eta R} = 0$$

$$-\frac{g'}{g}\eta - \frac{\alpha}{\beta} + \frac{Ug'}{g} + U' + \frac{U}{\eta} = 0$$

$$-\frac{\alpha}{\beta} + \frac{g'}{g}(U-\eta) + U' + \frac{U}{\eta} = 0 \qquad (3.31)$$

By equation (3.02)

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{hu}{r} = 0 \qquad (3.02)$$

$$h = \sqrt{\rho_0}\;\dot{R}\, H(\eta)$$

$$\frac{\partial h}{\partial t} = \dot{R}\left[ H\frac{1}{2}\rho_0^{-1/2}\frac{\partial \rho_0}{\partial t} + \frac{\partial H}{\partial t}\rho_0^{1/2}\frac{\partial \eta}{\partial t}\right] + \frac{\partial \dot{R}}{\partial t}\rho_0^{1/2} H \Bigg]$$

By equation (iii)

$$R = Bt^{\beta}$$

$$\dot{R} = \frac{\beta R}{t}$$

$$\ddot{R} = \frac{(\beta-1)}{\beta}\,\frac{\dot{R}^2}{R}$$

$$\frac{\partial \rho_0}{\partial t} = \rho_0 \frac{\alpha}{\beta}\frac{\dot{R}}{R}$$

$$\frac{\partial h}{\partial t} = \dot{R}\left[\frac{H}{2}\sqrt{\rho_0}\frac{\alpha}{\beta}\frac{\dot{R}}{R} + H'\sqrt{\rho_0}\left(\frac{-\eta}{R}\dot{R}\right)\right] + \frac{\partial \dot{R}}{\partial t}\sqrt{\rho_0}\, H$$

$$\frac{\partial h}{\partial t} = \frac{\dot{R}^2}{R}\sqrt{\rho_0}\left[\frac{H\alpha}{2\beta} - H'\eta\right] + \frac{(\beta-1)}{\beta}\frac{\dot{R}^2}{R}H$$

By equation (3.25)

$$h = \sqrt{\rho_0}\ \dot{R}\, H'(\eta)$$

$$\frac{\partial h}{\partial r} = \sqrt{\rho_0}\ \dot{R}\, H'\frac{\partial \eta}{\partial r}$$

$$\frac{\partial h}{\partial r} = \frac{\sqrt{\rho_0}\ \dot{R}\, H'}{R}$$

By equation (3.22)

$$u = \dot{R}\, U(\eta)$$

$$\frac{\partial u}{\partial r} = \frac{\dot{R}\, U'}{R}$$

substituting these values in equation (3.02) we get

$$\frac{\dot{R}^2}{R}\sqrt{\rho_0}\left[\frac{H\alpha}{2\beta} - H'\eta\right] + \dot{R}U\frac{\sqrt{\rho_0}\,\dot{R}}{R}H'$$
$$+\frac{(\beta-1)}{\beta}\frac{\dot{R}^2 H}{R} + \frac{\sqrt{\rho_0}\ \dot{R}H\,\dot{R}U'}{R} + \frac{\sqrt{\rho_0}\ \dot{R}H\,\dot{R}U'}{\eta R} = 0$$

$$\frac{H\alpha}{2\beta} - H'\eta + H'U + HU' + \frac{HU}{\eta} + \frac{(\beta-1)}{\beta}H = 0$$

$$\frac{\frac{\alpha}{2} + (\beta-1)}{\beta}H + H'(U-\eta) + HU' + \frac{HU}{\eta} = 0 \qquad (3.32)$$

By equation (3.03)

$$\frac{\partial u}{\partial t} + \frac{u\,\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{h^2}{\rho r} + \frac{GM}{r} - \frac{v^2}{r} = 0$$

By equation (3.22)

$$u = \dot{R}\, U(\eta)$$

$$\frac{\partial u}{\partial t} = \dot{R}\ U' \frac{\partial \eta}{\partial t} + U\ \ddot{R}$$

$$\frac{\partial u}{\partial t} = \frac{\dot{R}\, U'\, \eta}{R}\ \dot{R} + U\ \frac{(\beta-1)}{\beta}\ \frac{\dot{R}^2}{R}$$

and $\dfrac{\partial u}{\partial r} = \dfrac{\dot{R}U'}{R}$

By equation (3.24)

$$P = \rho_0\ \dot{R}^2\ X(\eta)$$

$$\frac{\partial P}{\partial r} = \rho_0\ \dot{R}^2\ X' \frac{\partial \eta}{\partial r}$$

$$\frac{\partial P}{\partial r} = \frac{\rho_0\ \dot{R}\ X'}{R}$$

By equation (3.25)

$$h = \sqrt{\rho_0}\ \dot{R}\ H(\eta)$$

$$\frac{\partial h}{\partial r} = \frac{\dot{R}\sqrt{\rho_0}\ H'}{R}$$

By equation (3.26)

$$v = \dot{R}\, V(\eta)$$

$$v^2 = \dot{R}^2\ V^2(\eta)$$

$$G = -\left[\frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\right] \frac{(2+\beta)(1+\beta)\dot{R}^2}{\pi \rho^{\alpha}\ r\beta+2}$$

$$m = m_1\ W$$

$$m = \frac{2\pi\ \rho^*\ r^{\beta+2}}{(2+\beta)}\ W$$

$$\frac{Gm}{r} = -\left[\frac{1}{\gamma M^2} + \frac{1}{2\ M_A^2}\right] \frac{(2+\beta)(1+\beta)\dot{R}^2}{\pi\ \rho^r\ r^{\beta+2}} \cdot \frac{2\pi\ \rho^*\ r(\beta+2)}{(2+\beta).\ \eta R} W$$

$$\frac{Gm}{r} = -\left[\frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\right] \frac{(1+\beta)\ 2W\ \dot{R}^2}{\eta\qquad R}$$

substituting these values in equation (3.03), we get

$$\frac{\dot{R}^2}{R}\left[U'\eta + U\frac{(\beta-1)}{\beta}\right] + \dot{R}U.\frac{\dot{R}}{R}U' + \frac{1}{\rho_0 g}\ \rho_0\ \frac{\dot{R}X'}{R} + \frac{\sqrt{\rho_0}\ \dot{R}H}{\rho_0\ g}\ \frac{\dot{R}}{R}\sqrt{\rho_0}\ H' + \frac{\rho_0\ \dot{R}^2 H^2}{\rho_0\ g\ \eta R}$$

$$\left[\frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\right]\frac{2(1+\beta)W}{\eta} - \frac{V^2\dot{R}^2}{\eta R} = 0$$

$$(U-\eta)U' + \frac{X'}{g} + \frac{H}{\eta g}[H+\eta H'] - \left[\frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\right]\frac{2(1+\beta)}{\eta}W - \frac{V^2}{\eta} = 0 \qquad (3.33)$$

By equation (3.04), we have

$$\frac{d}{dt}(vr) = 0$$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right](vr) = 0$$

$$\frac{\partial(vr)}{\partial t} + u\frac{\partial}{\partial r}(vr) = 0$$

$$v\frac{\partial r}{\partial t} + r\frac{\partial v}{\partial t} + ur\frac{\partial v}{\partial r} + vu = 0 \qquad \text{(a)}$$

By equation (3.26)

$$v = \dot{R}\ V(\eta)$$

$$\frac{\partial v}{\partial t} = \dot{R}V'\frac{\partial \eta}{\partial t} + V\ddot{R}$$

$$= -\ \dot{R}\ V'\frac{\eta\dot{R}}{R} + \frac{V(\beta-1)\dot{R}^2}{R\ B}$$

$$\frac{\partial v}{\partial t} = \frac{\dot{R}^2}{R}\left[-V'\eta + \frac{(\beta-1)}{\beta}V\right]$$

again equation (3.26)

$$v = \dot{R}\ V(\eta)$$

$$\frac{\partial v}{\partial r} = \dot{R}\ V'\frac{\partial \eta}{\partial r}$$

$$\frac{\partial v}{\partial r} = \frac{\dot{R}\ V'}{R}$$

$$\eta = \frac{r}{R(t)}$$

$$r = \eta R(t)$$

$$\frac{\partial r}{\partial t} = \eta \dot{R}$$

substituting these values in equation (a)

$$\dot{R}V\ \eta\dot{R} + \eta R\ \frac{\dot{R}^2}{R}\left[-\eta\ V' + V\ \frac{(\beta-1)}{\beta}\right] + \dot{R}\ U\ \eta R\ \frac{\dot{R}V'}{R} + \dot{R}\ V.\dot{R}\ U = 0$$

$$V\eta + \eta\left[-\eta V' + \frac{V\,(\beta-1)}{\beta}\right] + \eta U V' + UV = 0$$

$$U\,[V + \eta\ V'] - \eta\left[\frac{(\beta-1)}{\beta}\ V + \eta\ V'\right] = 0 \tag{3.34}$$

By equation (3.05)

$$\frac{\partial m}{\partial r} = 2\pi\rho r$$

$$m = m_1\ W(\eta)$$

$$\frac{\partial m}{\partial r} = m_1\ W'\ \frac{\partial \eta}{\partial r} + W\ \frac{\partial m_1}{\partial r}$$

$$= m_1\ \frac{W'}{R} + W\ \frac{2\pi\ \rho^*\ (2+\beta)}{r}\ r^{(\beta+1)}$$

$$= m_1\left[\frac{W'}{R} + \frac{W\ (2+\beta)}{\eta R}\right]$$

$$\frac{\partial m}{\partial r} = \frac{m_1}{R}\left[W^1 + \frac{W\,(2+\beta)}{\eta}\right]$$

Substituting putting these values in equation (3.05)

$$\frac{m_1}{R}\left[W' + W\ \frac{(2+\beta)}{\eta}\right] = 2\pi\ g\ R^{\beta}.\eta\ R$$

$$\frac{m_1}{R}\left[W' + W\ \frac{(2+\beta)}{\eta}\right] = g\ \frac{2\pi\rho^*\ r^{\beta+2}\eta^{\beta\rho^2}}{R\,(\beta+2)\eta}\ (\beta+2)$$

$$\frac{m_1}{R}\left[W' + W\ \frac{(2+\beta)}{\eta}\right] = \frac{m_1\,(2+\beta)}{R\eta}\ g$$

$$W' = \left[\frac{(2+\beta)g}{\eta} \quad \frac{(2+\beta)W}{\eta}\right] \tag{3.35}$$

By equation (3.06)

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{P}{\rho^2}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \frac{1}{\rho r}\frac{\partial}{\partial r}(jr) = 0$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{P}{\rho^2}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \frac{1}{\rho}\frac{\partial j}{\partial r} + \frac{j}{\rho r} = 0 \tag{b}$$

By equation (3.29)

$$e = \dot{R}^2\, E(\eta)$$

$$\frac{\partial e}{\partial t} = 2\dot{R}\,\ddot{R}\,E + \dot{R}^2\,E'\frac{\partial \eta}{\partial t}$$

$$= 2\dot{R}\frac{(\beta-1)}{\beta}\frac{\dot{R}^2}{R}E + \dot{R}^2\,E'\left(-\frac{\eta\dot{R}}{R}\right)$$

$$\frac{\partial e}{\partial t} = \frac{\dot{R}^3}{R}\left[\frac{2(\beta-1)}{\beta}E - \eta\,E'\right]$$

and

$$\frac{\partial e}{\partial r} = \dot{R}^2\;E'\frac{\partial \eta}{\partial r}$$

$$\frac{\partial e}{\partial r} = \frac{E'R^2}{R}$$

$$\frac{P}{\rho_\gamma}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] = \frac{\dot{R}^3}{g^2}\;\frac{X}{R}\left[-g'\eta + g\frac{\alpha}{\beta}\right]$$

$$j = J_0\,J(\eta)$$

$$\frac{\partial j}{\partial r} = \frac{J_0\,J'}{R}$$

Substituting these values in equation (b)

$$-\left[\frac{(\beta-1)}{\beta}E + \eta E'\right] + UE - \frac{P}{g^2}\left[\frac{\alpha}{\beta}g + (U-\eta)g'\right] + \frac{1}{Xg\gamma^{3/2}M^3}\left[J + X\,J'\right] = 0 \tag{3.36}$$

By equation (3.07)

$$\frac{\partial j}{\partial r} = kj$$

$$j = J_0\, J(\eta)$$

$$\frac{\partial j}{\partial r} = J_0\, J' \frac{\partial \eta}{\partial r}$$

$$\frac{\partial j}{\partial r} = \frac{J_0\, J'}{R}$$

$$\frac{J_0 J'}{R} = k_0\, \rho^n\, j^q\, r^s\, t^l$$

$$P_0^{3/2}\, \rho^{1/2} J' = k_0 \rho^\eta\, P^m\, j^q\, r^s t^l$$

$$J' = \alpha_1\, \eta^s\, g^n\, X^m\, J^{q+1} \tag{3.37}$$

where $\alpha_1 = k_0\, \rho_0^{n+m-1}\, j^{q+1}\, \dot{R}^{2m-2+s}$

The boundary conditions are by equation (3.09) & (3.14)

$$P_1 = \frac{2}{\gamma+1}\, \rho_0\, \dot{R}^2$$

$$\rho_0\, \dot{R}^2\, X(\eta) = \frac{2}{(\gamma+1)}\, \rho_0\, \dot{R}^2$$

at $\eta = 1$

$$X(1) = \frac{2}{(\gamma+1)} \tag{3.38}$$

By condition (3.20) & (3.23)

$$\rho_1 = \frac{\gamma+1}{\gamma-1}\, \rho_0$$

$$\rho_0\, g(\eta) = \frac{(\gamma+1)}{(\gamma-1)}\, \rho_0$$

at $\eta = 1$

$$g(1) = \frac{(\gamma+1)}{(\gamma-1)} \tag{3.39}$$

By condition (3.11) & (3.25)

$$h_1 = \frac{(\gamma+1)}{(\gamma-1)}\, h_0$$

$$\sqrt{\rho_0}\ \dot{R} H(\eta) = \frac{(\gamma+1)}{(\gamma-1)} \frac{\sqrt{\rho_0}\ \dot{R}}{mM_A}$$

at $\eta = 1$ where $M_A^2 = \dfrac{\rho_0 \dot{R}^2}{h_0^2}$

$$H_{(1)} = \frac{(\gamma+1)}{(\gamma-1)} \frac{1}{M_A} \tag{3.40}$$

By condition (3.12) & (3.22)

$$u_1 = \frac{2}{(\gamma+1)} \dot{R}$$

$$\dot{R}\, U(\eta) = \frac{2}{(\gamma+1)} \dot{R}$$

at $\eta = 1$

$$U(1) = \frac{2}{(\gamma+1)}$$

By condition (3.13) & (3.26)

$$v_1 = \frac{2}{(\gamma+1)} \dot{R}$$

$$\dot{R}\, V(\eta) = \frac{2}{(\gamma+1)} \dot{R}$$

at $\eta = 1$

$$V(1) = \frac{2}{(\gamma+1)} \tag{3.42}$$

By condition (3.14) & (3.29)

$$e_1 = \frac{2\dot{R}^2}{(\gamma+1)^2}$$

$$\dot{R}^2 E(\eta) = \frac{2\dot{R}^2}{(\gamma+1)^2}$$

at $\eta = 1$

$$E(1) = \frac{2}{(\gamma+1)^2} \tag{3.43}$$

By condition (3.15)

$$m_1 = \frac{2\pi \rho^x r^{(2+\beta)}}{(2+\beta)}$$

$$m_1 W(\eta) = m_1$$

$$\eta = 1$$

$$W(1) = 1 \tag{3.44}$$

By condition (3.28)

$$J_1 = J_0 J(\eta)$$

$$J_0 = J_0 J(\eta)$$

at $\eta = 1$

$$J(1) = 1 \tag{3.45}$$

And appropriate transformed boundary conditions with shock front are

$$X(1) = \frac{2}{(\gamma+1)} \tag{3.38}$$

$$g(1) = \frac{(\gamma+1)}{(\gamma-1)} \tag{3.39}$$

$$H(1) = \frac{(\gamma+1)}{(\gamma-1)} \frac{1}{M_A} \tag{3.40}$$

$$U(1) = \frac{2}{(\gamma+1)} \tag{3.41}$$

$$V(1) = \frac{2}{(\gamma+1)} \tag{3.42}$$

$$E(1) = \frac{2}{(\gamma+1)^2} \tag{3.43}$$

$$W(1) = 1 \tag{3.44}$$

$$J(1) = 1 \tag{3.45}$$

## RESULT AND DISCUSSION

The set of differential equation (3.31) – (3.37) have been integrated numerically with the help of boundary conditions (3.38) – (3.45) by well known Runga-kutta method for $\alpha_1 = 2$ and $2.5$ $\beta = 1.5$ & $-2$, $\gamma = 4/3,\ 7/5$ ; $M^2 = 5$, $M_A^2 = 10$, $\alpha = \dfrac{2(2-\gamma)}{(\gamma+1)}$ and $n = -1/2$, $M = 3/2$ $q = 0$, $s = 1$, we have plotted the graphs showing the variations of various flow parameters with distance for different values of $\gamma$, $\beta$ and $\alpha$, in pesence and absence of gravitational field and in presence and absence of radiation. This helped us to study the importance of gravitation and radiation respectively on flow parameters.

From Fig. (3.11) to Fig. (3.18) we observe that the radial component of velocity, magnetic field, energy, rotational velocity, radiation flux decrease as we go towards the center of the explosion, while density and pressure increase as we go towards the center of the shock. In Fig. (3.18), it is surprising to note that the mass decreases for the case of $\gamma = 7/5$ for both presence and absence of gravitational field, it remains uniform for the case of $\gamma = 4/3$. In absence of gravitational field decrease in the radial velocity, magnetic field and energy is more prominent. In the presence of gravitational field we see that decrease in rotational velocity, radiation flux is more prominent while the increase in the value of density and pressure is more prominent.

To draw a comparison between gravitational effects vis-a-vis rotational effect, we have also observed the variations of flow parameters in the absence of rotation in fig. (3.19) to fig. (3.25) and compared it with the earlier drawn graphs for the absence of gravitation. We find that radial velocity, magnetic field, energy decrease more in presence of rotational velocity while radiation flux decreases more rapidly in the absence of rotational velocity. As expected, in the absence more rapidly in the absence of rotational velocity. As expected, in the absence of rotation, the value of pressure and density increase more as we go towards the center of the shock. However, the peculiar phenomenon to be observed is that the variation of mass remains uniform in case of $\gamma = 4/3$, $\alpha_1 = 2$ and $\beta = 2$ irrespective of the presence or absence of the rotational

effect. We further note that the gravitational effect is important for the propagation of shock waves in the present problem.

We have calculated our result in suitable form

$$\frac{u}{u_1} = \frac{(\gamma+1)}{2} U(\eta)$$

$$\frac{P}{P_1} = \frac{(\gamma+1)}{2} X(\eta)$$

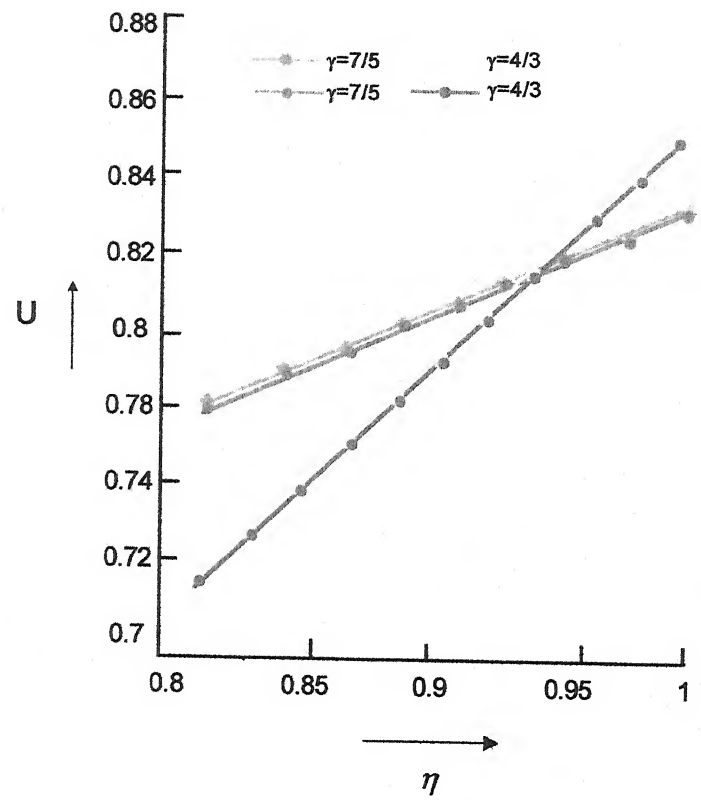
$$\frac{\rho}{\rho_1} = \frac{(\gamma-1)}{(\gamma+1)} g(\eta)$$

$$\frac{h}{h_1} = \frac{(\gamma-1)}{(\gamma+1)} M_A H(\eta)$$

$$\frac{e}{e_1} = \frac{(\gamma+1)^2}{2} E(\eta)$$

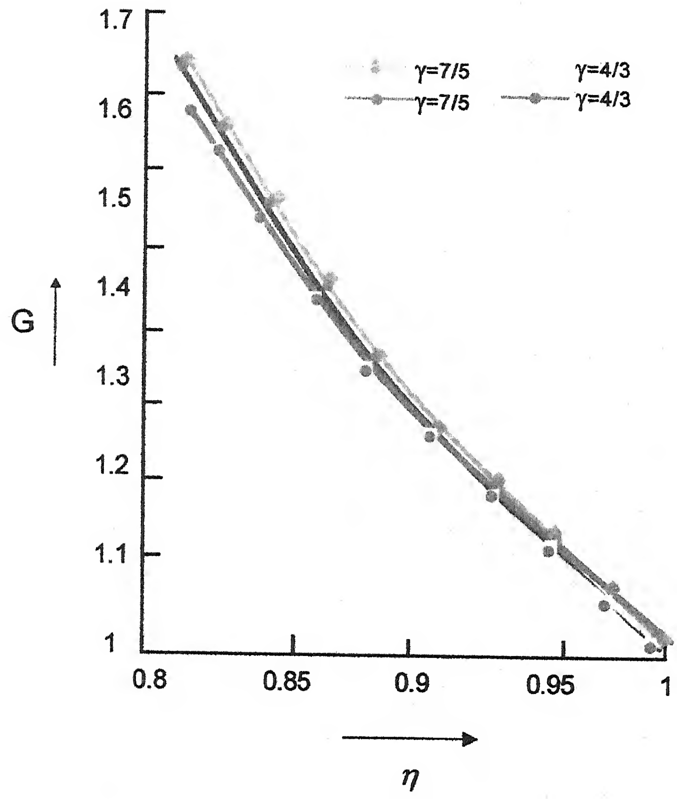
$$\frac{v}{v_1} = \frac{(\gamma+1)}{2} V(\eta)$$

$$\frac{m}{m_1} = W(\eta)$$



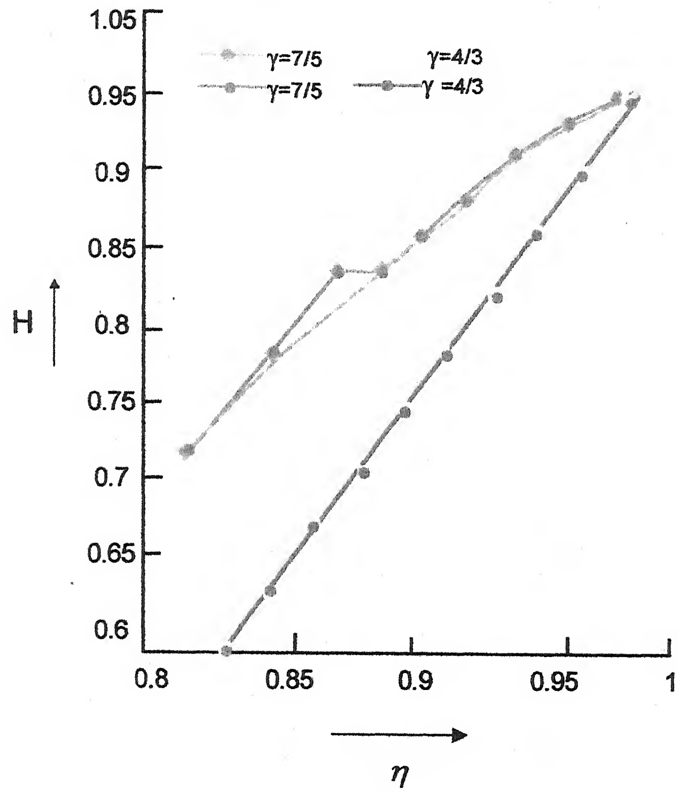

Variation of Radial velocity with Distance series 1&2 shows with Gravitation ($\gamma$=7/5, 4/3) & series 3&4 without Gravitation ($\gamma$=7/5, 4/3)

Fig. 3.01



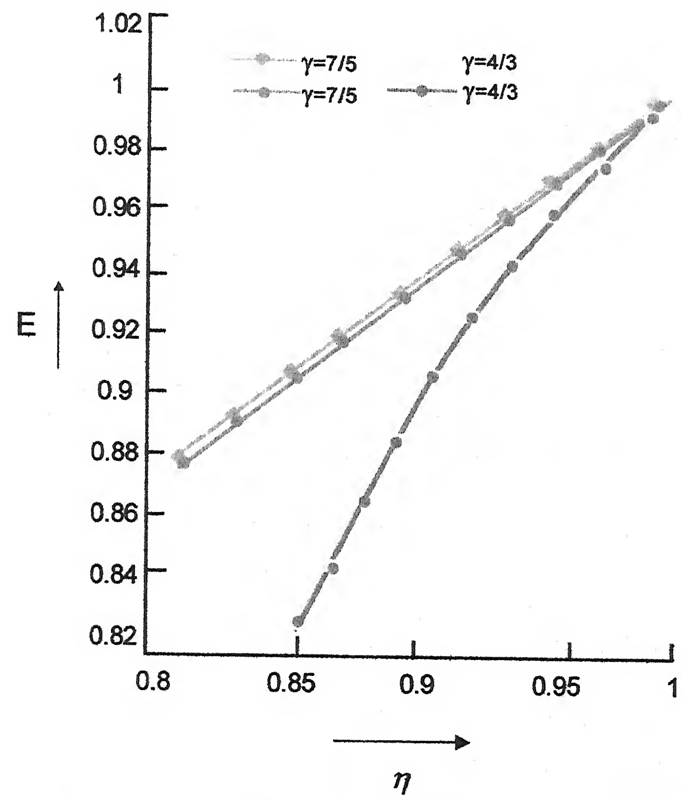

Variation of Density with Distance series 1&2 shows with Gravitation ($\gamma$=7/5, 4/3) & series 3&4 without Gravitation ($\gamma$=7/5, 4/3)

Fig. 3.02



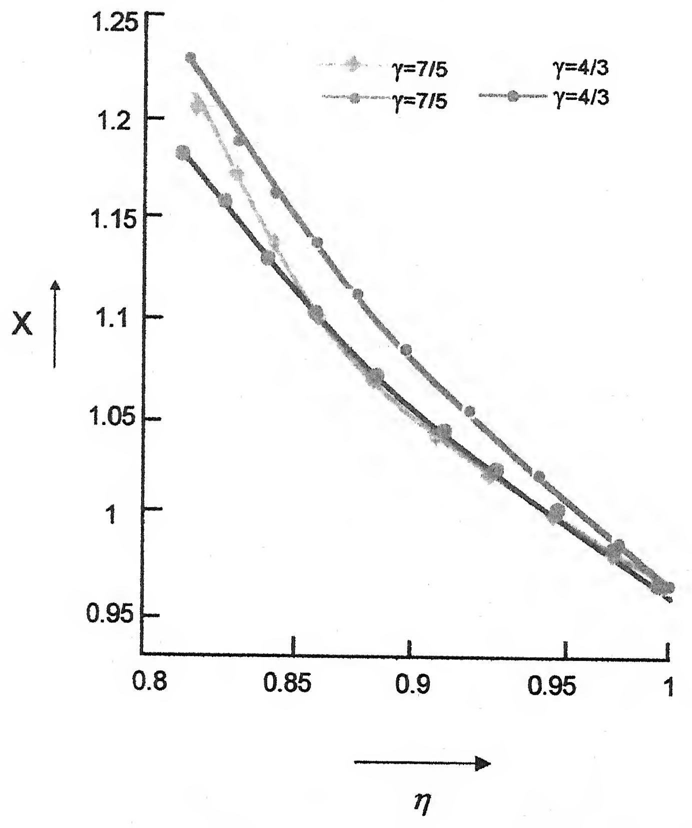

Variation of Magnetic field with Distance series 1&2 shows with Gravitation ($\gamma$=7/5, 4/3) & series 3&4 without Gravitation ($\gamma$=7/5, 4/3)

Fig. 3.03



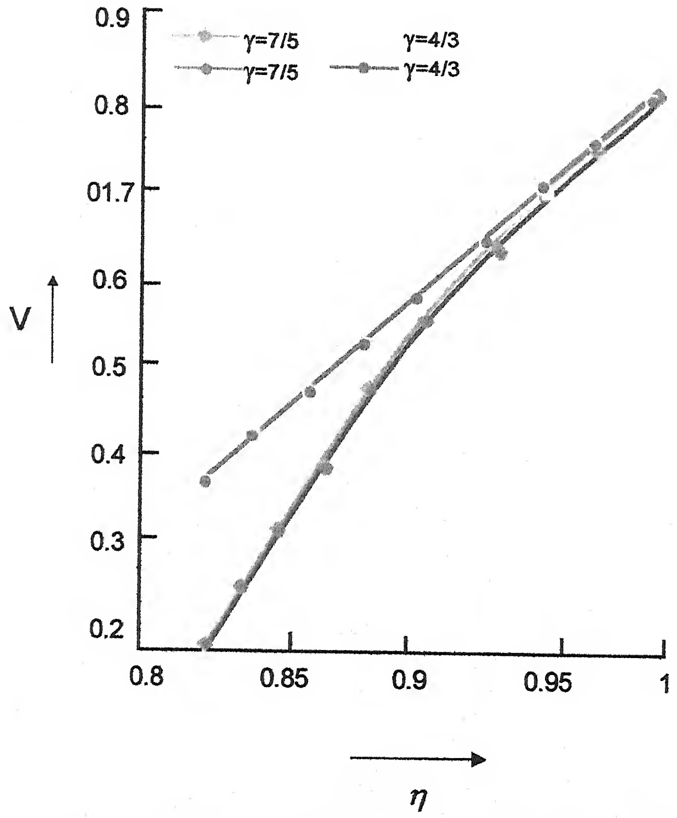

Variation of Magnetic field with Distance series 1&2 shows with Gravitation ($\gamma$=7/5, 4/3) & series 3&4 without Gravitation ($\gamma$=7/5, 4/3)

Fig. 3.04



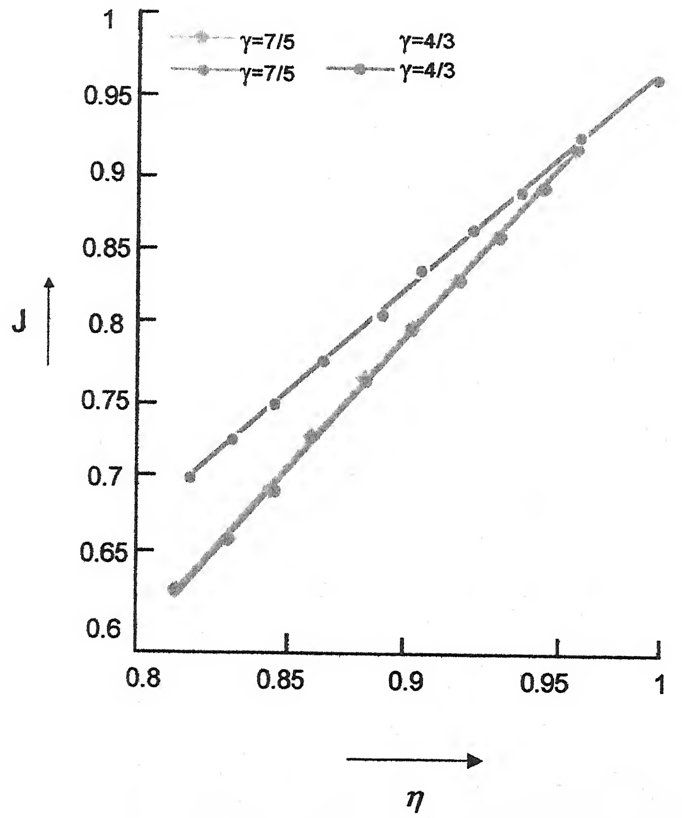

Variation of Pressure with Distance series 1&2 shows with Gravitation (γ=7/5, 4/3) & series 3&4 without Gravitation (γ=7/5, 4/3)

Fig. 3.05



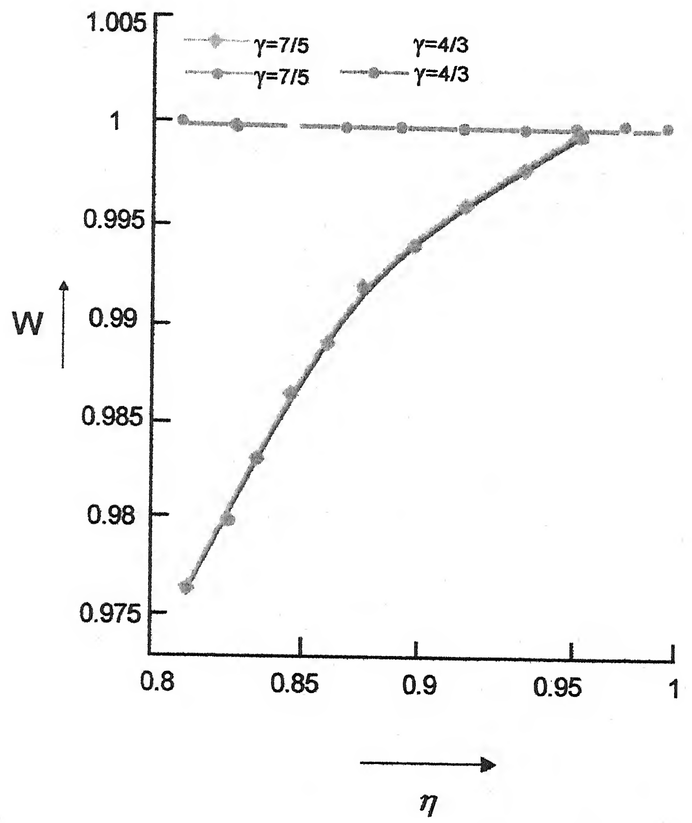

Variation of Rotational with Distance series 1&2 shows with Gravitation (γ=7/5, 4/3) & series 3&4 without Gravitation (γ=7/5, 4/3)

Fig. 3.06



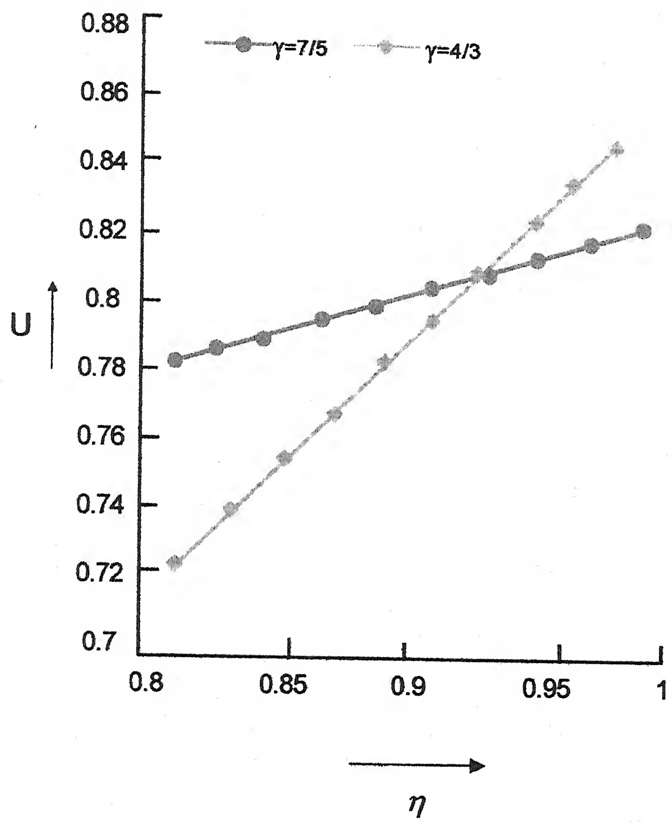

Variation of Radiation flux with Distance series 1&2 shows with Gravitation ($\gamma$=7/5, 4/3) & series 3&4 without Gravitation ($\gamma$=7/5, 4/3)

Fig. 3.07



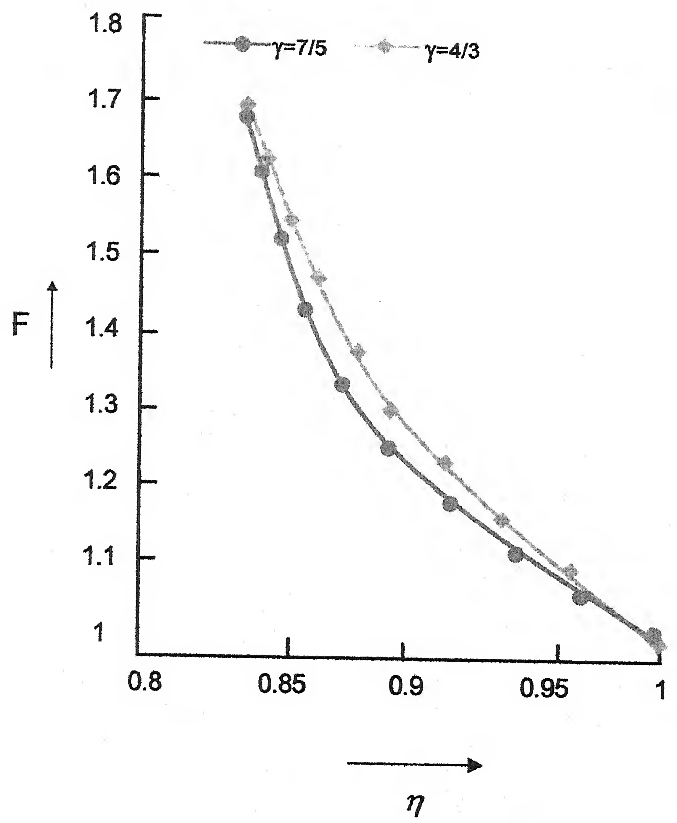

Variation of Mass with Distance series 1&2 shows with Gravitation ($\gamma$=7/5, 4/3) & series 3&4 without Gravitation ($\gamma$=7/5, 4/3)

Fig. 3.08




Variation of Radial velocity with Distance

Fig. 3.09




Variation of Density with Distance

Fig. 3.10

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# Chapter - IV

# *Propagation of plane shock wave in magnetogasdynamics*

## INTRODUCTION

The phenomena associated with heat transfer in rotating fluids are extremely complex whether the fluid is at rest or in motion, study or unsteady. Wang [1] considered the piston problem with thermal radiation for one dimensional unstudy shocks using the similar method of Sedov [2], Helliwel [3] took a more general case of piston problem with radiation heat transfer for general optically and transparent limit. Elliot [4] discussed the self similar solution for spherical blast waves in air using the Rossland diffusion approximation under the assumption that there is no effect of heat flux at the centre of symmetry laumnch and Probrteim [5] have studied the radiation effects of shock waves in an spherical medium several other author e.g. Summer [6]. Magnetic field also be taken into account. The unsteady model of Roche Consits of a gas distributed with spherical symmetry around a nucleous of large mass. It is assumed that the gravitating effect of the gas neglected compared with the attraction of the heavy nucleus Rosenau [7] [8].

In this chapter we consider propagation of plane shock wave under isothermal condition. In the isothermal condition temperature gradient becomes zero behind the shock and radiation effect are already implicity involved. An idealized magnetic field is considered for only a portion of sphere including the origin i.e. the point of explosion. According to Summer [6] the field is directly tangential to the advancing shock front. The shock is assumed to advance into conducting gas of decreasing density and produced by a decreasing magnetic field the total energy of the above is non-constant. Similarity solution for shock waves phenomena in magnetogasdynamic have been obtained by no of authors e.g. Levin [10] J.P. Vishwakarma [11], Shilpa Shinde [12], Michaut Haut et al. [13].

## EQUATION OF MOTION FOR ADIABATIC FLOW

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} = 0 \tag{4.01}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{h}{\rho} \frac{\partial h}{\partial r} + \frac{h^2}{\partial r} + \frac{Gm}{r^2} = 0 \tag{4.02}$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \gamma p \frac{\partial \rho}{\partial r} + (\gamma - 1) \frac{\partial f}{\partial r} = 0 \tag{4.03}$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + h \frac{u}{r} = 0 \tag{4.04}$$

where u, p, $\rho$, h and f are the velocity, pressure, density, magnetic field and radiative heat flux at a radial distance r from the center of the core at time t, m denotes the constant mass of the core, G be the gravitational constant, the magnetic permeability of the medium has taken to unity.

The equation of state for ideal gas is given by

$$P = \Gamma \rho T \tag{4.05}$$

Where $\Gamma$ is the gas constant.

Also assuming local thermodynamics equilibrium and taken Rossland's diffusion approximation, we have

$$F = \frac{C\mu}{3} \frac{\partial}{\partial r} \left(\sigma T^4\right) \tag{4.06}$$

Where (1/4) ac is the Stefan – Baltz man constant, C is the velocity of light and $\mu$ the mean free path of radiation, is a function of density and absolute temperature T, following Wang K.C. [1]

$$\mu = \mu_0 \rho^\alpha T^\beta \tag{4.07}$$

where $\mu_0$, $\alpha$ and $\beta$ being constant, the total energy of the wave is dependent on time as

$$E = E_0 t^q \qquad (0 < q < 4/3) \tag{4.08}$$

Where $E_0$ being the constant in the front of shock wave, we assume that

$$\rho_1 = AR^{-\omega} \qquad (0 \le \omega \le 2) \tag{4.09}$$

where A is constant and R denotes the radius of the shock surface ahead the shock, the magnetic field distribution is assumed to be as

$$h_1 = CR^{-k} \qquad (2K = \omega + 1) \tag{4.10}$$

At the equilibrium state the other flow variable ahead the shock are

$$U_1 = 0, \qquad \rho_1 = \left[\frac{AmG}{1+\omega}\right] R^{-(1+\omega)} \frac{C^2}{K\,(1+k)\,R^{-2k}} \tag{4.11}$$

where C, W and K are constant the disturbance is heated by an isothermal shock and Ranking Hugoniot jump conditions are.

$$\rho_2 \left(V - u_2\right) = \rho_1 \; v = m_s \tag{4.12}$$

$$p_2 \; \frac{h_2^2}{2 - p_1} - \frac{h_2^2}{2} = M_s \, . \, u_2 \tag{4.13}$$

$$h_2 \left(v - u_2\right) = h_1 \; v \tag{4.14}$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}\left(V - u_2\right)^2 \frac{h_2^2}{2} - \frac{F_2}{M_s} = e_1 + \frac{p_1}{\rho_1} + \frac{v_2}{2} + \frac{h_1^2}{\rho_1} \tag{4.15}$$

$$T_2 = T_1 \tag{4.16}$$

Where suffix 2 denote the flow variable just behind the and V be the velocity of the shock and is given by

$$V = \frac{dR}{dt} \tag{4.17}$$

**SOLUTION OF THE EQUATION OF MOTION**

In order to reduce the equation of flow to ordinary differential equation, we now introduce the following similarity transformation Sedov [2]

$$\eta = \left(VmG\right)^{-1/2} \, rt^{-\delta} \tag{4.18}$$

where

$$\delta = \frac{2}{3} = \frac{(2+q)}{(5-\omega)}$$

i.e.

$$q = \frac{2}{3}\ (2-\omega) \tag{4.19}$$

We see the solution of equation (4.04 – 4.19) in the form

$$U = \frac{r}{t}\, v\ (\eta)$$

$$\rho = \frac{(Am\, Gt^2)}{r^{\omega+3}}\, D\ (\eta)$$

$$p = \frac{(AmG)}{r^{\omega+3}}\ (\eta)$$

$$h = \sqrt{\ }\ \frac{(Am\, G)}{r^{(\omega+1)2}}\ N\ (\eta)$$

$$f = \frac{(Am\, G)}{r^{\omega}}\ Q(\eta)$$

Where $\eta$ is constant that assume the value 1 at the shock front hence we have

$$R = (vm\, G)^{1/3}\ t^{2/3} \tag{4.20}$$

By use of equation (4.18) in equation (4.07) with the add of equation (4.05) we obtain

$$\alpha = \frac{\omega}{\omega+1} \quad \text{and} \quad \beta = \frac{(5\omega+7)}{2(\omega+1)} \tag{4.21}$$

Equation (4.01-4.04) and equation 6 are then transformed with the relation (4.18) and (4.20) to the form.

$$\frac{du}{d\eta} = p\ \frac{[Q(v-\delta)]}{(D^{\alpha-\beta-4}\ \rho^{\beta+4})} - \omega\, v + \frac{2\{v\delta+1\}-\{\omega+1\}(v-\delta)+(v+\delta)}{\eta^2 v}\ x$$

$$\frac{\eta\left[(v-\delta)^2\ D - P - N^2 + N^2\left[v(3-\omega)-(v-\delta)(1-\omega)+v(v-1)(v-\delta)D\right]\right.}{\eta\left[(v+\delta)^2\ D - P - N^2\, D\right]} \tag{4.22}$$

$$\frac{dD}{d\eta} = \frac{D}{(v-\delta)}\ \frac{[(\omega v-2)]}{\eta} - \frac{dv}{d\eta} \tag{4.23}$$

$$\frac{dp}{d\eta} = p\,\frac{1}{\eta}\left\{\frac{(\omega v - 2)}{v-\delta} - \frac{Q}{LD^{\alpha-\beta-4}\,p^{\beta+4} - 2}\right\} - \frac{1}{(v-\partial)}\,\frac{dv}{d\eta} \tag{4.24}$$

$$\frac{dN}{d\eta} = -\frac{N}{(v-\delta)}\left[\frac{v(3-\omega)}{2\eta} + \frac{dv}{d\eta}\right] \tag{4.25}$$

$$\frac{dQ}{d\eta} = \frac{\gamma p}{(\gamma-1)\eta}\left[\frac{(v-\delta)(2+Q)}{LD^{\alpha-\beta-4}\,p^{\beta+4}} - \frac{v}{\gamma}\,\{\omega(\gamma-1)+3\gamma-1\}+2\right] - \frac{Q}{\eta}\,(2-\omega) + \frac{dp}{d\eta}(\gamma-\delta) \tag{4.26}$$

$$\text{where}\quad L = \left(\frac{4ca\,\mu_0}{31^{\beta+4}}\right)(AGM)^{\alpha-1} \tag{4.27}$$

is a dimensionless radiation parameter.

## EQUATION OF MOTION FOR ISOTHERMAL FLOW

The equation of continuity, momentum and magnetic field are the same as in the adiabatic flow but the energy equation becomes

$$T = \text{Cons}\tan t$$

$$\frac{\partial T}{\partial r} = 0 \tag{4.28}$$

By use of equation 4.05, we have

$$\frac{P}{P_2} = \frac{\rho}{\rho_2} \tag{4.29}$$

By substitution of relation (4.18), (4.20) and (4.22) in equation (4.01-4.03) and (4.27) the following set of equation are obtained.

$$\frac{dv}{d\eta} = p\,\frac{[p(1-v-\delta)+(v-\delta)(1+\omega)-v]+N^2X[v(3-\omega)-(1+\omega)(v-\delta)]}{\{\eta(v-\delta)^2 D - N^2 - P\}}$$

$$+\;\frac{(v-\delta)\left[v(v-1)+\dfrac{1}{\eta^2 v}\right]}{\{\eta\,(v-\delta)^2 D - N^2 - P\}} \tag{4.30}$$

$$\frac{dD}{d\eta} = \frac{D}{(v-\delta)}\left[\frac{1}{\eta(\omega v-2)} - \frac{dv}{d\eta}\right] \tag{4.31}$$

$$\frac{d\rho}{d\eta} = p\left[\left(\frac{1}{D}\right)\frac{dD}{d\eta} - \frac{2}{\eta}\right] \tag{4.32}$$

$$\frac{dN}{d\eta} = -\frac{N}{(v-\delta)}\left[\left(\frac{v}{\eta}\right)\frac{(3-\omega)}{2} + \frac{dv}{d\eta}\right] \tag{4.33}$$

For the isothermal flow we also assume that the jump condition are heated by isothermal flow as given by equation (4.12-4.16).

## RESULT AND DISCUSSION

The transformed jump condition at the shock for both adiabatic and isothermal flow are

$$V(1) = \left(\frac{2}{3}\right)\Omega \tag{4.34}$$

$$D(1) = \frac{v}{(1-\Omega)} \tag{4.35}$$

$$P(1) = \frac{4v}{9}\left[\frac{\Omega+1}{\gamma M^2} + \frac{\Omega(2-\Omega)M_A^2}{\{2(1-\Omega)^2\}}\right] \tag{4.36}$$

$$N(1) = \frac{2}{3}\sqrt{\ }\, v\left(\frac{1}{1-\Omega}\right)M_A' \tag{4.37}$$

and

$$Q(1) = \frac{8v}{37}\left[\Omega\left\{\frac{(1-\Omega)\gamma M^2 - 1}{(\gamma-1)M^2}\right\} + \left\{\frac{M_A^2(\gamma\Omega-2)\Omega}{2(1-\Omega)(\gamma-1)} + \frac{(2-\Omega)\Omega}{2}\right\}\right] \tag{4.38}$$

Where V is obtained from the relation

$$V = \left(\frac{9}{2}\right) vM^2\left[2(1+\Omega) - \gamma\left(\frac{M^2}{M_A}\right)(1-\Omega)\right]^{-1} \tag{4.39}$$

For exhibiting the numerical solution, it is convenient to write variable in non dimensional form as,

$$\frac{u}{u_2} = \eta\left[\frac{v(\eta)}{\eta(1)}\right] \tag{4.40}$$

$$\frac{\rho}{\rho_2} = \frac{1}{\eta^{\omega+1}}\left[\frac{p(\eta)}{p(1)}\right] \tag{4.41}$$

$$\frac{p}{p_2} = \frac{1}{\eta^{\omega+1}}\left[\frac{p(\eta)}{p(1)}\right] \tag{4.42}$$

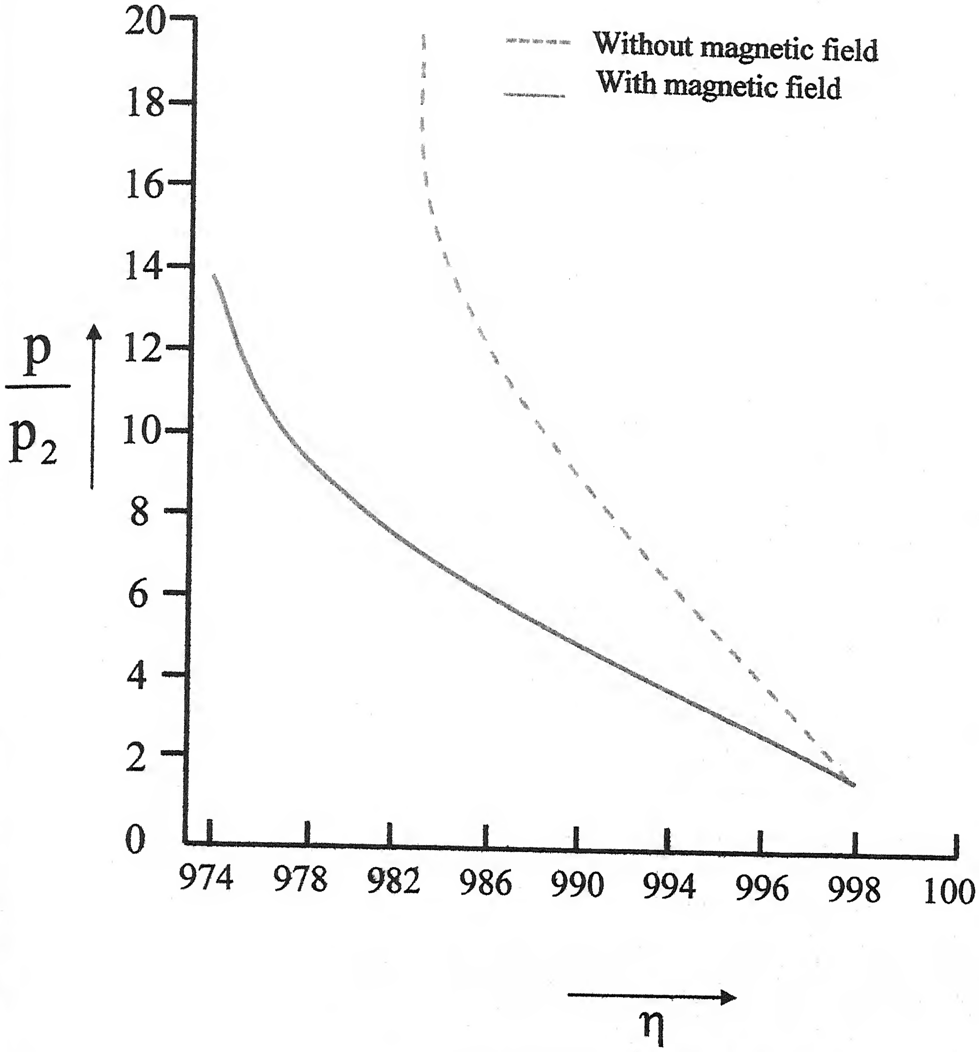
$$\frac{h}{h_2} = \frac{1}{\eta\,(1+\omega)^2}\left[\frac{N(\eta)}{N(1)}\right] \tag{4.43}$$

and

$$\frac{F}{F_2} = \frac{1}{\eta^{\omega}}\left[\frac{Q(\eta)}{Q(1)}\right] \tag{4.44}$$

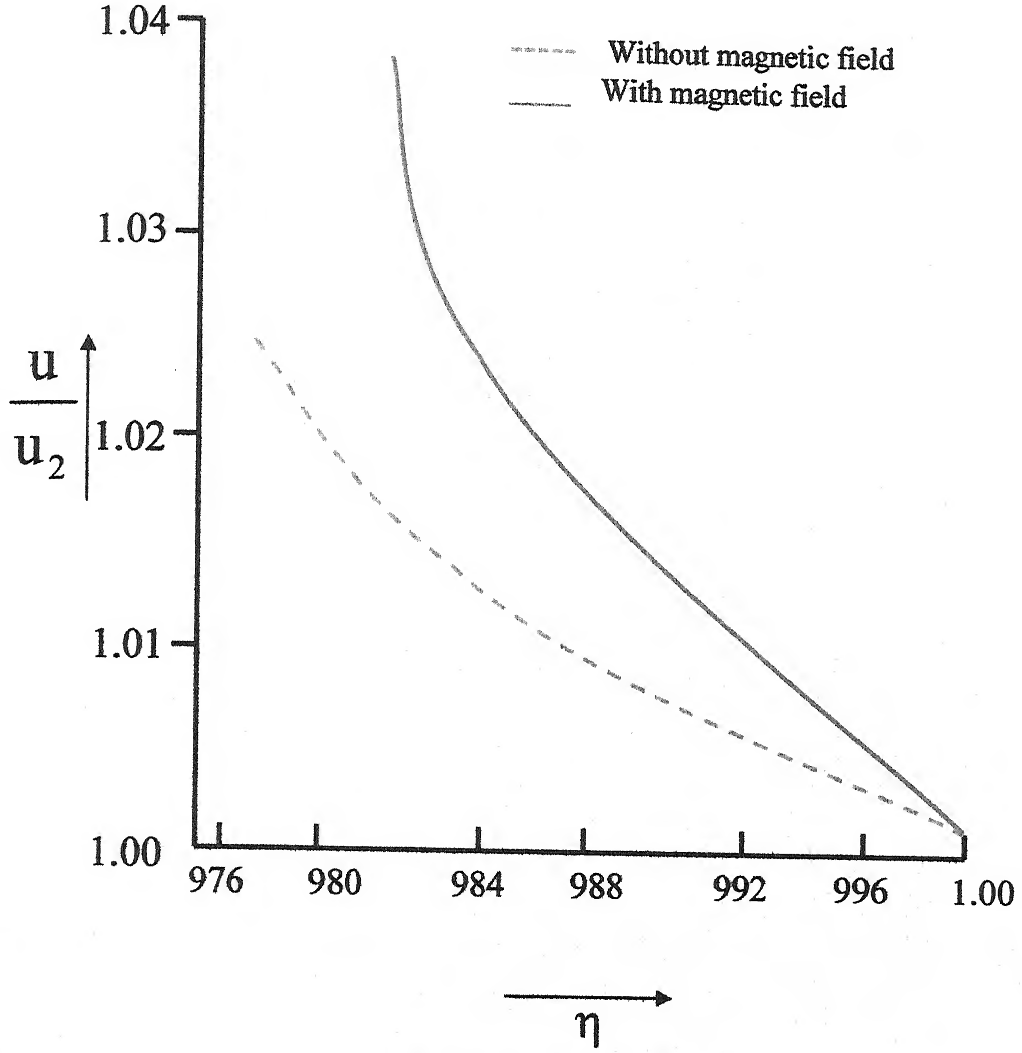
The numerical integration is carried out for adiabatic and isothermal flow separately on DEC-system 1090 computer installed at IIT Kanpur by the well known RKG system programme for the there values $\omega = 1.00$, $\omega = 1.50$ and $\omega = 2.00$. The other constants are $M^2 = 20$, $M_A^2 = 10$, $\delta = 2/3$, and $L=10$.

The nature of flow and field variable for both the adiabatic and isothermal cases are illustrated through figure (4.01-4.05) it is clear from the figure that velocity, density, pressure and magnetic field are maximum at the shock front and decreases rapidly towards the centre of explosion, when the flow behind the shock wave is isothermal but these variables are minimum at the shock front and increase steadly so we move towards the centre of explosion in adiabatic case. The condition of radiation heat flux has been illustrated in figure 5 we therefore conclude that the flow and field parameters are being subsidied, when the assumption of adiabatic to be not valid and the temperature gradient of the flow becomes zero.



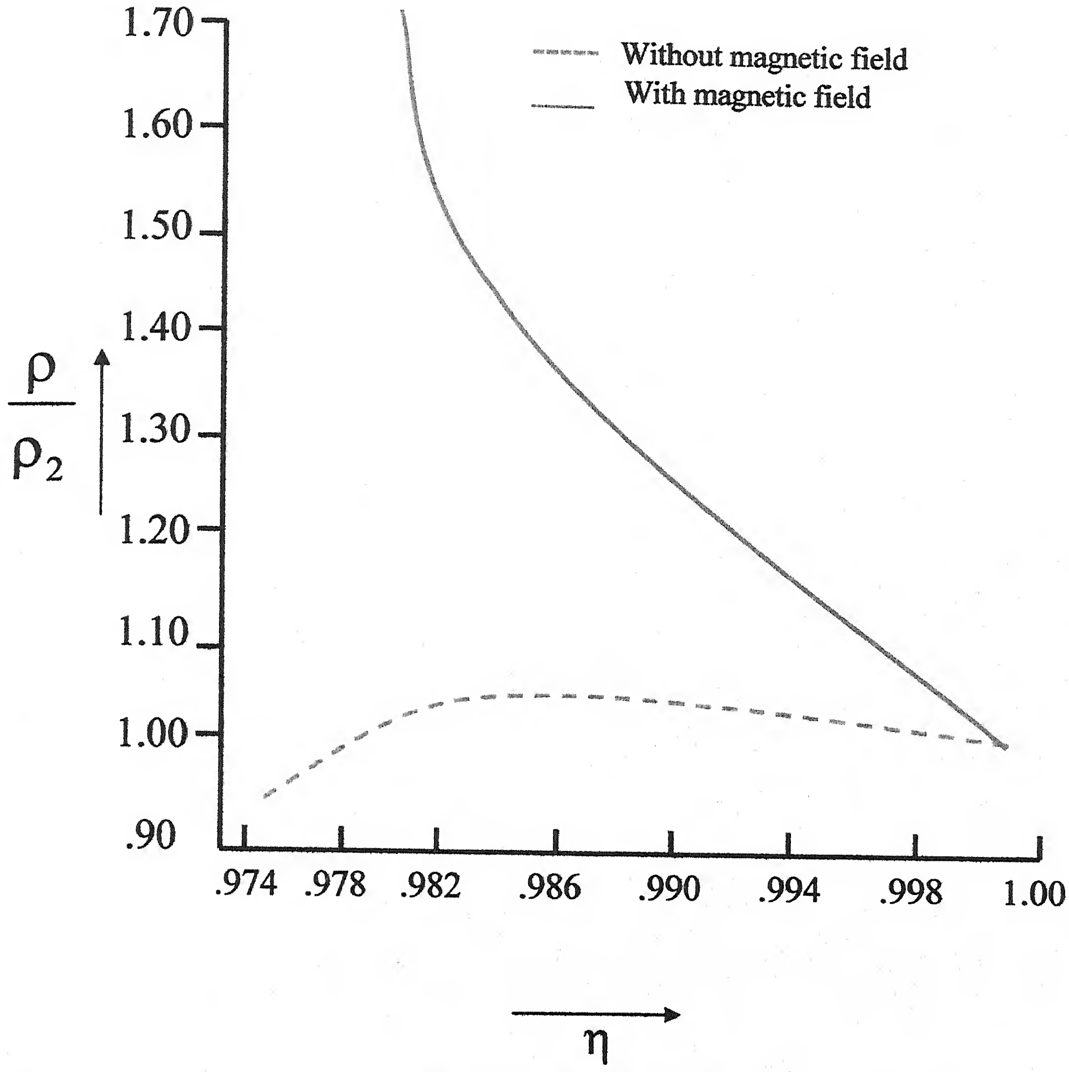

**Pressure Distribution**

**Fig. 4.1**



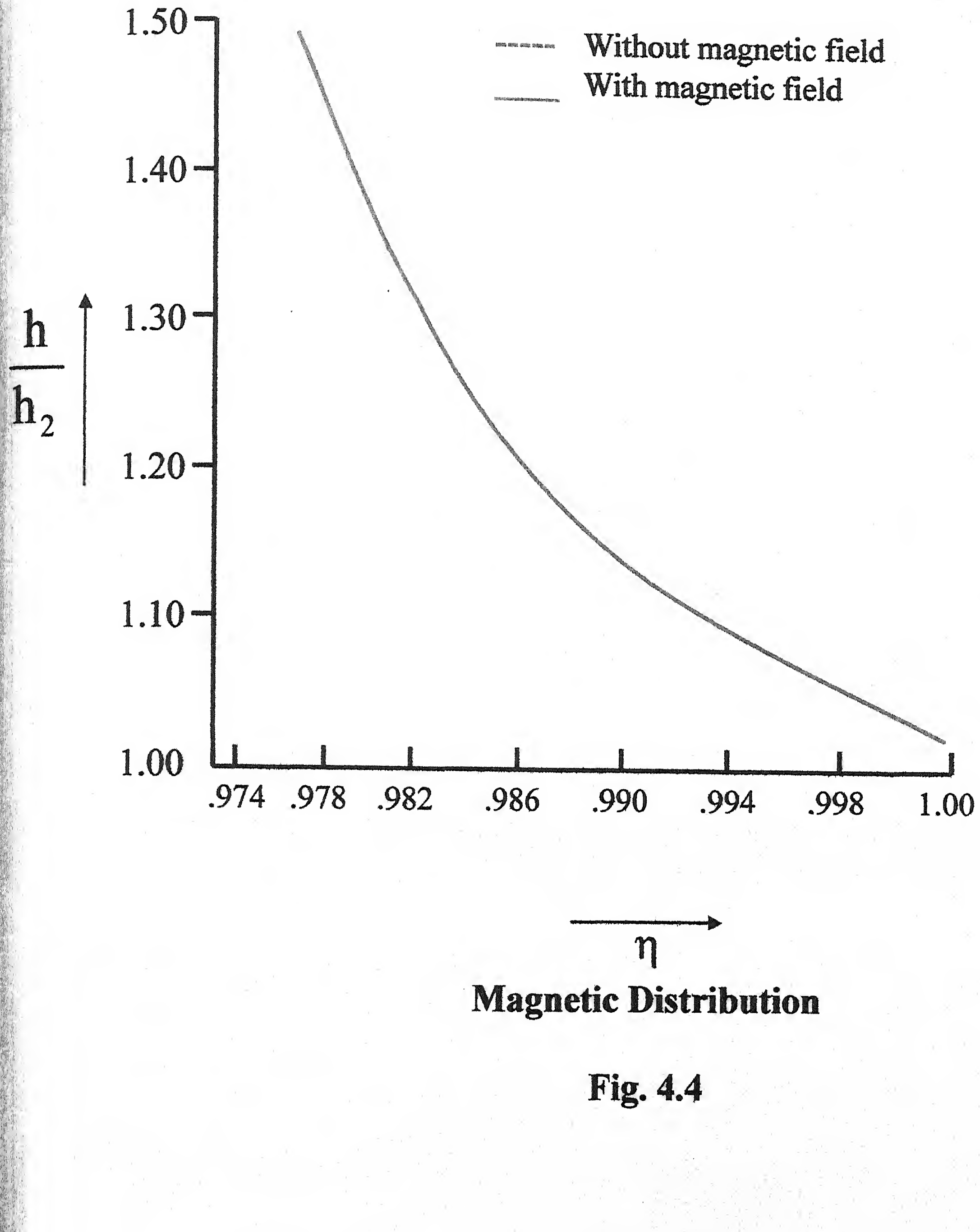

**Velocity Distribution**

**Fig. 4.2**




**Density Distribution**

**Fig. 4.3**




**Magnetic Distribution**

**Fig. 4.4**

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# Chapter - V
# *Self Similar Solution of Cylindrical Shock Wave in Magnetogasdynamics.*

## INTRODUCTION

Self similar solution for shock wave have studied by kopal and Lin [1] using similarity method of sedov [2] Elliot [3] wang [4] Helliwell [5] have considered the problem of shock wave with thermal radiation. Self similar solution for adiabatic flow and is thermal flow was obtained by Rao and Ramana [6] Ojha [7] studied the same problem having effects of azimuthal magnetic field. Singh [8] has discussed the self similar adiabatic flow of self gravitating gas in ordinary gas. Srivastava [9] is also studied self similar solution for magnetic shock wave. G.J. Ball [10] D. Kh ofengein has discussed simulation of blast wave propagation over a cylinder & Purohit [11] R.F. Chishell [12] have obtain an analytic description of converging shock waves D. Bhardwaj [13] has been obtain the result formation of shock waves in magnetogas dynamic flows. S. Sreekanth, A.S. Nagarajan and S. Venkata Ramana [14] have obtain transient MHD free convection flow of on incompressible viscous dissiputive fluid. S. Ahmed and N. Ahmed [15] have recently obtain two dimentional MHD oscillatory flow along a uniformally moving infinite vertical parlous plate bounded by porous medium.

Present chapter deals self-similar flow of cylindrical weak shock wave taking gravitational forces under non uniform atmosphere taking radiation heat flux into account. The radiation pressure and radiation energy have been ignored we have considered the problem shock wave reduced by a point explosion in a gas.

We have considered the problem M. Cheng, K.C. Hung, O.Y. Chong [16] has discussed on numerical study of water mitigation affection blast wave Koichi Noguchi, Edison Liang [17] has discussed on three dimensional particle acceleration in electromagnetic cylinder and torus. Vishwakarma and Yadav [18] studied the solution for self similar flow behind a magnetogasdynamics shock wave in a radiative.

In this chapter we show that the magnetic field has a significant effect gas is self gravitating. The motion of the shock wave is assumed to satisfy the power law

$$\rho_0 = AR^{-\alpha} \tag{5.01}$$

Where A and $\alpha$ are constant and R is the shock radius. The total energy of the flow increases with time because of the pressure exerted on the gas by an expanding surface therefore.

$$E = \beta t^q \ (q > o) \tag{5.02}$$

Where $\beta$ and q are constant and E is the total energy. The magnetic field distribution law is

$$h_0 = CR^{-\beta} \ (\beta \geq 0) \tag{5.03}$$

Where C and $\beta$ are constant and the values of q & $\beta$ are to be determined later. The magnetic field is directed tangential to advancing shock front.

The flow variable just ahead of the shock are

$$u_0 = 0,\ m_0 = \frac{4\pi A}{(3-\alpha)} R^{(3-\alpha)}$$

$$\& \qquad P_0 = \frac{2\pi A^2 G}{(\alpha-1)(3-\alpha)} R^{2-2\alpha} + \frac{c^2}{2\beta}(1-\beta)R^{-2\beta} \tag{5.04}$$

where $1+\beta = \alpha$ and viscosity is neglected.

We investigate three types of models, the first having total energy of the explosion to be constant $(\alpha = 2.5)$, the second having constant velocity of propogation of shock waves $(\alpha = 2)$ and the third having neither constant total energy nor constant velocity propagation of shock waves $(\alpha = 1.5)$. An idealized magnetic field is considered for only a portion of sphere enclosing the origin i.e. the point of explosion.

## EQUATION OF THE PROBLEM

The basic differential equation governing. The adiabatic flow in a self gravitating gas are given by

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial t} + \frac{u}{r}\right) = 0 \tag{5.05}$$

$$\frac{Du}{Dt}+\frac{1}{\rho}\frac{\partial}{\partial r}\left(P+P_r\right)+\frac{h_z}{\rho}\frac{\partial h_z}{\partial r}+\frac{h_z^2}{\rho r}=0 \tag{5.06}$$

$$\frac{Dh_z}{Dt}+h_z\frac{\partial u}{\partial r}+h_z\frac{u}{r}=0 \tag{5.07}$$

$$\frac{D}{Dt}h_\theta+h_\theta\frac{\partial u}{\partial r}+\frac{h_\theta u}{r}=0 \tag{5.08}$$

$$\frac{D}{Dt}\left(\frac{P}{\rho^\gamma}\right)=0 \tag{5.09}$$

$$\frac{\partial m}{\partial r}=4\pi\rho r^2 \tag{5.10}$$

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial r} \tag{5.11}$$

Where r, t, m, u, $\rho$, P, $\gamma$, $P_r$, $h_\theta$, $h_z$ and G are radial distance from the centre, time, mass contained in a sphere of radius $\gamma$, velocity, density, pressure and ratio of two specific heats, materials pressure, azumuthal magnetic field, tranverse magnetic field respectively. G represents gravitational constant.

The Rankine Huggoniot boundary conditions at the shock (5.09) are

$$u_1=\frac{\xi-1}{\xi}V \tag{5.12}$$

$$\rho_1=\rho_0\,\xi \tag{5.13}$$

$$P_1=\Psi\rho_0V^2 \tag{5.14}$$

$$P_{r_1}=\Psi\rho_0V^2 \tag{5.15}$$

$$h_{\theta_1}=h_{\theta 0}\,\xi \tag{5.16}$$

$$h_{z_1}=h_{z0}\,\xi \tag{5.17}$$

$$m_1=m_0 \tag{5.18}$$

Where

$$\psi=\frac{1}{\gamma m^2}+\frac{2(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[\frac{1}{M^2}+\frac{(\gamma-1)}{4M_A^2}(\xi-1)^2\right] \tag{5.19}$$

$v = \frac{dR}{dt}$ being the shock velocity and $\xi$ is given by the quadratic equation

$$M_A^{-2}(2-\gamma)\xi^2 + \left[(2\gamma-1)+\frac{2}{M^2}\right]\xi-(\gamma-1)=0 \tag{5.20}$$

The mach number M and Alfen's mach number $M_A$ are given by $M^2 = \frac{V^2\rho_0}{\gamma P_0}$ and

$$M_A^2 = \frac{V^2\rho_0}{h_{\theta_0}^2} \tag{5.21}$$

Let us seck the solution to the equation in the following form.

$$u = \frac{r}{t}\,U(\eta) \tag{5.22}$$

$$\rho = r^k\, t^\lambda\, \Omega(\eta) \tag{5.23}$$

$$m = r^{k+3}\, t^\lambda\, W(\eta) \tag{5.24}$$

$$P = r^{k+2}\, t^{\lambda-2}\, P(\eta) \tag{5.25}$$

$$P_r = r^{k+2}\, t^{\lambda-2}\, P_r(\eta) \tag{5.26}$$

$$h_\theta = r^{(k+2)/2}\, t^{(\lambda-2)/2}\, N_\theta(\eta) \tag{5.27}$$

$$h_z = r^{k+2/2}\, N_z(\eta) \tag{5.28}$$

Where $\eta = r^a t^b$ while k, $\lambda$, a and b are constants and are to be determined from the conditions of problem. The total energy E inside the shock wave of radius R is given by

$$E = 4\pi\int_0^R \left(\frac{1}{2}\rho u^2 + \frac{P}{(\gamma-1)} + \frac{h^2}{2} - \frac{Gm\rho}{r}\right) r^2\, dr = B t^q \;(q \geq 0) \tag{5.29}$$

In terms of the variables $\eta$, we can express the total energy as

$$E = \frac{4\pi}{a}\int_\eta^{\eta_0}\left[\eta^{[(k+5)/a]}\, r^{\lambda-2-b/a(k-5)}\left(\frac{1}{2}U^2\Omega+\frac{P}{(\gamma-1)}+N\right) - GW\Omega\eta^{[(2K+5)/a]-1}\right.$$
$$\left. t^{2\lambda-(b/a)(2k+5)}\right]d\eta = Bt^q \tag{5.30}$$

Equation (5.29) yields

$$\frac{a}{b} = \frac{-5}{(4+q)} \tag{5.31}$$

We choose $\eta_0$ to be constant at the shock surfaces. This choice fixes the velocity of the shock as

$$V = -\frac{b}{a}\frac{R}{t} \tag{5.32}$$

Using equations (5.21 to 5.23) in equation (5.13) to (5.19) and in equation (5.21) we assume, without any loss off generality.

$$K = -\alpha,\ \lambda = 0,\ b = 4+q,\ a = -5 \text{ and } \alpha = \frac{10}{4+q} \tag{5.33}$$

**SOLUTION OF EQUATION OF MOTIONS**

The equation (2.17 to 2.18) of are used to reduce the equation

$$\frac{\partial\rho}{\partial T} + u\frac{\partial\rho}{\partial r} + \frac{\rho\partial u}{\partial r} + \frac{\rho u}{r} = 0$$

By equation (5.17 & 5.18)

$$\rho = r^k t^\lambda \, \Omega(\eta)$$

$$u = \frac{r}{t} \, U(\eta)$$

$$\eta = r^a t^b$$

$$\frac{\partial\eta}{\partial t} = b t^{b-1} r^a = \frac{b t^b r^a}{t}$$

$$\frac{\partial\eta}{\partial t} = \frac{b\eta}{t}$$

and $\dfrac{\partial\eta}{\partial r} = a r^{a-1} t^b = \dfrac{a(r^a t^b)}{r}$

$$\frac{\partial\eta}{\partial t} = \frac{b\eta}{t}$$

$$\frac{\partial\eta}{\partial r} = \frac{a\eta}{r}$$

$$u = \frac{r}{t} U(\eta)$$

$$\frac{\partial u}{\partial r} = \frac{1}{t}\left[r U' \frac{\partial \eta}{\partial r} + U \frac{\partial \eta}{\partial r}\right]$$

$$= \frac{1}{t}\left[r U' \frac{a\eta}{r} + U\right]$$

$$\frac{\partial u}{\partial r} = \frac{1}{t}\left[U' a\eta + U\right]$$

$$\rho = r^k \, t^\lambda \, \Omega(\eta)$$

$$\frac{\partial \rho}{\partial t} = r^k\left[t^\lambda \, \Omega' \frac{\partial \eta}{\partial t} + \Omega \, \lambda t^{\lambda-1}\right]$$

$$= r^k\left[t^\lambda \, \Omega' \frac{b\eta}{t} + \frac{\Omega\lambda t^\lambda}{t}\right]$$

$$\frac{\partial \rho}{\partial t} = \frac{r^k t^\lambda}{t}\left[\Omega^1 \, b\eta + \lambda\Omega\right]$$

&

$$\frac{\partial \rho}{\partial r} = t^\lambda\left[k r^{k-1} \, \Omega + r^K \, \Omega' \frac{\partial \eta}{\partial r}\right]$$

$$= t^\lambda\left[\frac{k r^k}{r} \, \Omega + r^K \, \Omega' \frac{a\eta}{r}\right]$$

$$\frac{\partial \rho}{\partial r} = \frac{t^\lambda r^k}{r}\left[k\Omega + \Omega^1 \, a\eta\right]$$

putting values $u, \frac{\partial \rho}{\partial t}, \frac{\partial \rho}{\partial r}, \frac{\partial u}{\partial r}$ in equation we get

$$\frac{r^k t^\lambda}{t}\left[\lambda\Omega + \Omega' \, \eta b\right] + \frac{r}{t} U\left[k\Omega + \Omega' \, a \, \eta\right] . r^{k-1} t^\lambda + \frac{r^k t^\lambda \Omega}{t}\left[U + U' a\eta\right] + r^k t^\lambda \, \Omega \frac{r}{t} \, u \, \frac{1}{r} = 0$$

$$\left[\lambda\Omega + \Omega' \, \eta b\right] + U\left[k\Omega + \Omega' a\eta\right] + \Omega\left[U + U' a\eta\right] = 0$$

Using equation (5.32) in above equation

$$\frac{10}{\alpha}\Omega'\,\eta-\alpha\,\Omega\,U-5\,\Omega'\,\eta U+U\Omega-5U'\,\eta\Omega+\Omega U=0$$

$$5\Omega'\eta\left(\frac{2}{\alpha}-U\right)+(2-\alpha)\,U\,\Omega-5\eta\Omega U=0$$

$$\frac{5\Omega'}{\Omega}\eta\left(\frac{2}{\alpha}-U\right)+(2-\alpha)\,U-5\eta U'=0 \tag{5.34}$$

Using equation (5.06)

$$\frac{Du}{Dt}+\frac{1}{\rho}\left(P+P_r\right)+\frac{h_z}{\rho}\frac{\partial h_z}{\partial r}+\frac{h_z^{2}}{\rho r}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{1}{\rho}\frac{\partial P_r}{\partial r}+\frac{h_z}{\rho}\frac{\partial h_z}{\partial r}+\frac{h_z^{2}}{\rho r}=0 \tag{b}$$

$$u=\frac{r}{t}\,U\,(\eta) \tag{5.35}$$

$$\frac{\partial u}{\partial t}=r\left[U'\,\frac{\partial\eta}{\partial t}\,\frac{1}{t}+U\,\left(\frac{-1}{t^2}\right)\right]$$

$$\frac{\partial u}{\partial t}=r\left[\frac{b\eta}{t}\,\frac{1}{t}\,U'-\frac{U}{t^2}\right]$$

$$\frac{\partial u}{\partial t}=\frac{r}{t^2}\left[b\eta U'-U\right] \tag{v}$$

&

$$\frac{\partial u}{\partial r}=\frac{1}{t}\left[U'\,a\eta+U\right] \tag{ii}$$

By equation (5.24)

$$P=r^{k+2}\,t^{\lambda-2}\,P(\eta)$$

$$\frac{\partial P}{\partial r}=t^{\lambda-2}\left[(k+2)r^{k+1}.P(\eta)+r^{k+2}\,P'(\eta)\frac{\partial\eta}{\partial r}\right]$$

$$\frac{\partial P}{\partial r}=t^{\lambda-2}\left[(k+2)\,r^{k+1}\,P(\eta)+r^{k+2}.\,P'(\eta)\frac{a\eta}{r}\right]$$

$$t^{\lambda-2}\,r^{k+1}\left[(k+2)P(\eta)+a\eta P'(\eta)\right] \tag{vi}$$

&

$$\frac{\partial P_r}{\partial r} = t^{\lambda-2}\, r^{k+1}\left[(k+2)P_r\,(\eta)+a\,\eta P_r{}^{1}(\eta)\right] \tag{vii}$$

Using equation (5.27)

$$h_z = r^{(k+2)/2}\, t^{(\lambda-2)/2}\, Nz(\eta)$$

$$\frac{h_z}{\partial r} = t^{(\lambda-2)/2}\left[\frac{(k+2)}{2}\, r^{\left(\frac{k+2}{2}-1\right)} N_z(\eta) + r^{\frac{(k+2)}{2}}\,.\, N_z'(\eta)\frac{\partial \eta}{\partial r}\right]$$

$$= t^{\frac{(\lambda-2)}{2}}\left[\frac{(k+2)}{2}\, r^{\frac{k+2}{2}-1} N_z(\eta) + r^{\frac{k+2}{2}} N_z'(\eta)\frac{a\eta}{r}\right]$$

$$= t^{\frac{\lambda-2}{2}}\, r^{\frac{k}{2}}\left[\frac{(k+2)}{2}\, r^{\frac{k+2}{2}-1} N_z(\eta) + N_z'(\eta).a\eta\right] \tag{viii}$$

$$h_z^2 = \left(r^{\frac{k+2}{2}}\, t^{\frac{\lambda-2}{2}}\, N_z(\eta)\right)^2 \tag{ix}$$

putting the value of equation (5.21) & (v) to (ix) in equation (b), we get

$$\frac{r}{t^2}\left[b\eta\, U' - U\right] + \frac{r}{t}U\frac{1}{t}\left[b\eta\, U' - U\right] + \frac{1}{r^k t^\lambda}\,\Omega.\, t^{\lambda-2}.\, r^{k+1}\left[(k+2)P + a\,\eta\, P'\right]$$

$$+\frac{1}{r^k t^\lambda}\,\Omega\, t^{\lambda-2}\, r^{k+1}\left[(k+2)\,P_r + a\eta\, P_r'\right] + \frac{r^{\frac{k+2}{2}}\, t^{\frac{\lambda-2}{2}}\, N_z}{r^k\, t^\lambda\, \Omega}\; t^{\frac{\lambda-2}{2}}\, r^{\frac{k}{2}}\left[\frac{k+2}{2}N_z + N_z'\, a\eta\right]$$

$$+\frac{\left(r^{\frac{k+2}{2}}\, t^{\frac{\lambda-2}{2}}\, N_z\right)}{r^k\, t^\lambda\, \Omega.\, r} = 0$$

$$\left[b\eta\, U' - U\right] + U\left[b\eta U' - U\right]$$

$$+\Omega\left[(k+2)P + a\eta P'\right] + \Omega\left[(k+2)\,P_r + a\,\eta\, P_r'\right] + N_z\left[\frac{(k+2)}{2}N_z + N_z'\right] + N_z^2 = 0$$

substituting the value (5.32)

$$k = -\alpha,\ \lambda = 0 \qquad b = 4+q \qquad a = -5 \quad \& \quad \alpha = \frac{10}{4+q} \tag{5.36}$$

in above equation

$$\frac{10}{\alpha}\, U'\eta - U + U^2 - 5UU'\eta + \frac{(2-\alpha)}{\Omega}\, P - \frac{5P'\eta}{\Omega} + \frac{(2-\alpha)}{\Omega}\, P_r - \frac{5P_r'\eta}{\Omega} = 0 \tag{5.37}$$

Using equation (5.07)

$$\frac{Dh_z}{Dt} + h_z \frac{\partial u}{\partial r} + \frac{h_z U}{r} = 0$$

$$\frac{\partial}{\partial t} h_z + u \frac{\partial}{\partial r} h_z + h_z \frac{\partial u}{\partial r} + h_z \frac{u}{r} = 0 \quad (c)$$

$$u = \frac{r}{t} U(\eta) \tag{5.38}$$

$$\frac{\partial u}{\partial r} = \frac{1}{t}\left[U' a\eta - U\right] \tag{ii}$$

$$h_z = r^{\frac{k+2}{2}} t^{\frac{\lambda-2}{2}} N_z(\eta)$$

$$\frac{\partial h_z}{\partial t} = r^{\frac{k+2}{2}}\left[t^{\frac{\lambda-2}{2}} N_z'(\eta)\frac{\partial \eta}{\partial t} + N_z(\eta)\frac{\lambda-2}{2} t^{\frac{\lambda-2}{2}-1}\right]$$

$$\frac{\partial h_z}{\partial t} = r^{\frac{k+2}{2}}\left[t^{\frac{\lambda-2}{2}} N_z' \frac{b\eta}{t} + N_z t^{\frac{\lambda-4}{2}} \frac{(\lambda-2)}{2}\right]$$

$$\frac{\partial h_z}{\partial t} = r^{\frac{k+2}{2}} t^{\frac{\lambda-4}{2}}\left[N_z' b\eta + \frac{(\lambda-2)}{2} N_z\right] \tag{x}$$

By equation (viii)

$$\frac{\partial h_z}{\partial r} = t^{\frac{\lambda-2}{2}} r^{\frac{k}{2}}\left[\frac{k+2}{2} N_z + N_z' a\eta\right]$$

Putting these values in equation (c)

$$r^{\frac{k+2}{2}} t^{\frac{\lambda-4}{2}}\left[N_z' b\eta + \frac{(\lambda-2)}{2} N_z\right] + \frac{r}{t} U t^{\frac{\lambda-2}{2}} r^{\frac{k}{2}}\left[\frac{(k+2)}{2} N_z + N_z' a\eta\right]$$

$$+ r^{\frac{k+2}{2}} t^{\frac{\lambda-2}{2}} N_z \frac{1}{t}\left[U' a\eta - U\right] + r^{\frac{k+2}{2}} t^{\frac{\lambda-2}{2}} N_z \frac{r}{t} U \frac{1}{r} = 0$$

$$\left[N_z' b\eta + \frac{(\lambda-2)}{2} N_z\right] + U\left[\frac{k+2}{2} N_z + N_z' a\eta\right] + N_z\left[U' a\eta - U\right] + N_z U = 0$$

Substituting the values of equation (5.32)

$K=-\alpha,\ \lambda=0,\ b=4+q,\ a=-5$ and $\alpha=\dfrac{10}{4+q}$

$$\frac{10}{\alpha}N_z'\eta-N_z+U\frac{(2-\alpha)}{2}N_z-5\eta N_z'U+N_z\left[-5U'\eta-U\right]+N_zU=0$$

$$5\eta N_z'\left(\frac{2}{\alpha}-U\right)-5U'\eta N_z+(U-1)N_z-\frac{\alpha}{2}UN_z=0-$$

$$5\frac{N_z'}{N_z}\eta\left[\frac{2}{\alpha}-U\right]-5U'\eta+(U-1)-\frac{\alpha}{2}U=0$$

Now, by using (5.08)

$$\frac{D}{Dt}h_\theta+h_\theta\frac{\partial u}{\partial r}+h_\theta\frac{u}{r}=0$$

$$\frac{\partial}{\partial t}h_\theta+u\frac{\partial h_\theta}{\partial r}+h_\theta\frac{\partial u}{\partial r}+\frac{h_\theta u}{r}=0 \qquad \text{(d)}$$

$$u=\frac{r}{t}U(\eta) \qquad (5.21)$$

$$h_\theta=r^{\frac{k+2}{2}}t^{(\lambda-2)/2}N_\theta(\eta)$$

$$\frac{\partial h_\theta}{\partial t}=r^{\frac{k+2}{2}}\left[\frac{(\lambda-2)}{2}t^{\frac{\lambda-2}{2}-1}N_\theta(\eta)\right]+t^{\frac{\lambda-2}{2}}N_\theta'(\eta)\frac{\partial\eta}{\partial t}$$

$$=r^{\frac{k+2}{2}}\left[\frac{\lambda-2}{2}t^{\frac{\lambda-4}{2}}N_\theta(\eta)+t^{\frac{\lambda-2}{2}}N_\theta'(\eta)\frac{b\eta}{t}\right]$$

Since $\qquad \dfrac{\partial\eta}{\partial t}=\dfrac{b\eta}{t}$

$$\frac{\partial h_\theta}{\partial t}=r^{\frac{k+2}{2}}\left[\frac{\lambda-2}{2}t^{\frac{\lambda-4}{2}}N_\theta(\eta)+t^{\frac{\lambda-2}{2}-1}N_\theta'(\eta)b\eta\right]$$

$$\frac{\partial h_\theta}{\partial t}=r^{\frac{k+2}{2}}t^{\frac{\lambda-4}{2}}\left[\frac{\lambda-2}{2}N_\theta(\eta)+b\eta N_\theta'(\eta)\right] \qquad \text{(xi)}$$

$$h_\theta=r^{\frac{k+2}{2}}t^{\frac{\lambda-2}{2}}N_\theta(\eta)$$

$$\frac{\partial h_\theta}{\partial r}=t^{\frac{\lambda-2}{2}}\left[r^{\frac{k+2}{2}}N_\theta'(\eta)\frac{a\eta}{r}+\frac{k+2}{2}r^{k/2}N_\theta(\eta)\right]$$

$$\frac{\partial\eta}{\partial r}=\frac{a\eta}{r} \qquad \text{(xii)}$$

Substituting the values of equation (5.21 ), (5.26) (ii), (xi) (xii) in equation (d), we get

$$r^{\frac{k+2}{2}}\ t^{\frac{\lambda-4}{2}}\left[N_\theta^{'}\, b\eta+\frac{(\lambda-2)}{2}N_\theta\right]+\frac{r}{t}U\,t^{\frac{\lambda-2}{2}}\ r^{\frac{k}{2}}\left[\frac{k+2}{2}N_\theta+N_\theta^{'}\ a\eta\right]$$

$$+r^{\frac{k+2}{2}}\ t^{\frac{\lambda-2}{2}}\ \frac{N_\theta}{t}\left[U^{'}\ a\eta-U\right]+r^{\frac{k+2}{2}}\ t^{\frac{\lambda-2}{2}}\ N_\theta\ \frac{r}{t}\ U\ \frac{1}{r}=0$$

$$\left[N_\theta^{'}\ b\eta+\frac{(\lambda-2)}{2}N_\theta\right]+U\left[\frac{k+2}{2}N_\theta+N_\theta^{'}\ a\eta\right]+N_\theta\ \left[U^{'}a\eta-U\right]+N_\theta\ U=0$$

substituting the following values from equation (5.32)

$$k=-\alpha,\ \lambda=0,\ b=4+q,\ a=-5\ \&\ \alpha=\frac{10}{4+q}$$

We get

$$\frac{10}{\alpha}N_\theta^{'}\ \eta-N_\theta+\frac{U(2-\alpha)}{2}N_\theta-5\eta\ N_\theta^{'}\ U+N_\theta\ \left[-5U^{'}\eta-U\right]+N_\theta\ U=0$$

$$5\eta N_\theta^{'}\left(\frac{2}{\alpha}-U\right)-5U^{'}\eta\ N_\theta+(U-1)\,N_\theta-\frac{\alpha}{2}U\ N_\theta\ =0$$

$$5\eta\ \frac{N_\theta^{'}}{N_\theta^{'}}\ \left(\frac{2}{\alpha}-U\right)-5U^{'}\eta+(U-1)-\frac{\alpha}{2}U=0$$

Now, using equation (5.09)

$$\frac{D}{Dt}\left(\frac{P}{\rho^{\gamma}}\right)=0$$

$$\frac{\partial}{\partial t}\left(\frac{P}{\rho^{\gamma}}\right)+u\ \frac{\partial}{\partial r}\left(\frac{P}{\rho^{\gamma}}\right)=0$$

$$\frac{1}{\left(\rho^{\gamma}\right)^{2}}\left[\rho^{\gamma}\ \frac{\partial P}{\partial t}-\gamma\,\rho^{\gamma-1}\ \frac{\partial\rho}{\partial t}.\ P+u\,\rho^{\gamma}\ \frac{\partial P}{\partial r}-P\ \gamma.\rho^{\gamma-1}\ \frac{\partial\rho}{\rho r}\right]=0$$

$$\rho^{\gamma}\ \frac{\partial P}{\partial t}-\gamma\,\rho^{\gamma-1}\ \frac{\partial\rho}{\partial t}.\ P+u.\,\rho^{\gamma}\ \frac{\partial P}{\partial r}-P.\gamma.\rho^{\gamma-1}\ \frac{\partial\rho}{\partial r}\ =0$$

$$\frac{\partial P}{\partial t}-\gamma\ \frac{\partial\rho}{\partial t}\ \frac{P}{\rho}+u\ \frac{\partial P}{\partial r}-\frac{P}{\rho}\gamma\ \frac{\partial\rho}{\partial r}=0 \qquad \text{(c)}$$

By equation (5.24)

$$P = r^{k+2}\ t^{\lambda-2}\ P(\eta)$$

$$\frac{\partial P}{\partial t} = r^{k+2}\left[t^{\lambda-2}\ P'\ \frac{\partial \eta}{\partial t} + (\lambda-2)\, t^{\lambda-3}\ P\right]$$

$$= r^{k+2}\left[t^{\lambda-2}\ P'\ \frac{b\eta}{t} + (\lambda-2)\, P\, t^{\lambda-3}\right]$$

$$\frac{\partial P}{\partial t} = r^{k+2}\, t^{\lambda-3}\left[(\lambda-2)\, P + P'\, b\eta\right] \qquad \text{(xiii)}$$

Again by equation (5.24)

$$P = r^{k+2}\ t^{\lambda-2}\ P(\eta)$$

$$\frac{\partial P}{\partial r} = t^{\lambda-2}\left[r^{k+2}\ P'\ \frac{\partial \eta}{\partial r} + P\,(k+2)\, r^{k+1}\right]$$

$$= t^{\lambda-2}\left[r^{k+2}\ P'\ \frac{a\eta}{r} + (k+2) P\, r^{k+1}\right]$$

$$\frac{\partial P}{\partial r} = t^{\lambda-2}\ r^{k+1}\left[P'\, a\,\eta + (k+2) P\right] \qquad \text{(xiv)}$$

By equation (iv)

$$\frac{\partial \rho}{\partial r} = \frac{t^{\lambda}\, r^{k}}{r}\left[k\,\Omega + \Omega'\ a\,\eta\right] \qquad \text{(iv)}$$

By equation (v)

$$\frac{\partial \rho}{\partial t} = \frac{r^{k}\, t^{\lambda}}{t}\left[\Omega'\ b\,\eta + \lambda\,\Omega\right] \qquad \text{(v)}$$

Putting values of equation (iv), (v) (xiii), (xiv) in equation (c)

$$r^{k+2}\ t^{\lambda-3}\left[(\lambda-2) P + P'\, b\,\eta\right] - \gamma\ \frac{r^{k}\, t^{\lambda}}{t}$$

$$\left[\Omega'\, b\,\eta + \lambda\,\Omega\right]\frac{r^{k+2}\ t^{\lambda-2}\ P}{r^{k}\ t^{\lambda}\,\Omega(\eta)}$$

$$+\frac{r}{t}\ U.\, t^{\lambda-2}\ r^{k+1}\left[P'\ a\,\eta + (k+2) P\right] - \frac{r^{k+2}\ t^{\lambda-2}\, P}{r^{k}.\ t^{\lambda}\ \Omega}\ \gamma\ \frac{t^{\lambda}\, r^{k}}{r}\left[k\,\Omega + \Omega'\ a\,\eta\right]$$

$$\left[(\lambda-2)\, P + P'\, b\eta\right] - \gamma\left[\Omega'\ b\eta + \lambda\,\Omega\right]\frac{P}{\Omega} + \left[(k+2)\, P + P'\, a\,\eta\right] - \gamma\left[k\,\Omega + \Omega^{1}\ a\,\eta\right]\frac{P}{\Omega} = 0$$

Now using equation (5.32)

$$k = -\alpha,\ \lambda = 0,\ a = -5\ \ b = 4+q,\ \alpha = \frac{10}{4+q}$$

We get,

$$\left[-2P + \frac{10}{\alpha} P' \eta\right] - \gamma P\left(\Omega' \eta \frac{10}{\alpha}\right) + (2-\alpha) U P - P' U (-5)\eta$$

$$-\gamma \left[P(-\alpha\Omega - 5\Omega'\eta)\right] U = 0$$

$$(2-\alpha)\, UP - \gamma P\left[\Omega' \frac{10}{\alpha} \eta - 5\Omega' \eta - \alpha\, \Omega U\right] + 5\eta\left(\frac{2}{\alpha} - U\right) P' = 0$$

$$5\eta \frac{P'}{P}\left(\frac{2}{\alpha} - U\right) + U(2-\alpha) - 2 - \gamma\left[-\alpha U + 5\eta \frac{\Omega'}{\Omega}\left(\frac{2}{\alpha} - U\right)\right] = 0$$

Now using equation (5.10)

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2 \qquad \text{(f)}$$

Taking values from equation (5.22) & (5.23)

$$\rho = r^k\, t^\lambda\, \Omega(\eta) \qquad (5.22)$$

$$m = r^{k+3}\, t^\lambda\, W(\eta) \qquad (5.23)$$

$$\frac{\partial m}{\partial r} = t^\lambda\left[(k+3) r^{k+2}\, W + r^{k+3}\, W' \frac{\partial \eta}{\partial r}\right]$$

$$= t^\lambda\left[(k+3) r^{k+2}\, W + r^{k+3}\, W' \frac{a\eta}{r}\right]$$

$$\frac{\partial m}{\partial r} = r^{k+2} . t^\lambda \left[(k+3)W + W' a\, \eta\right]$$

Putting these values is equation (f)

$$r^{k+2}\, t^\lambda \left[(k+3)W + W'\ a\eta\right] = 4\pi .\, r^k .\, t^\lambda .\, \Omega r^2$$

$$(k+3)\, W + W'\ a\,\eta = 4\pi\Omega$$

$$(k+2)W + W'\ a\,\eta - 4\pi\, \Omega = 0$$

$$\lambda = 0,\ k = -\alpha,\ a = -5,\ b = 4+q\ \alpha = \frac{10}{4+q}$$

$$(3-\alpha)\, W - 5W' - 4\pi\, \Omega = 0$$

$$5\eta\frac{W'}{W}+\frac{4\pi\Omega}{W}-(3-\alpha)=0$$

where

$$\overline{\Omega}' = \frac{\Omega'}{A},\quad \overline{P}' = \frac{P'}{A},\quad \overline{N}' = \frac{N}{A},\quad \overline{W}' = \frac{W'}{A} \qquad \text{(xv)}$$

$$\overline{\Omega} = \frac{\Omega}{A},\quad \overline{P} = \frac{P}{A},\quad \overline{N} = \frac{N}{A},\quad \overline{W} = \frac{W'}{A}$$

$$L = \frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)}M_A^{-2} \qquad \text{(xvi)}$$

The boundary condition (5.12) to (5.18) are transformed into the following form

Boundary condition (5.12) is

$$u_1 = \frac{\xi-1}{\xi}V$$

$$\frac{r}{t}U(\eta) = \frac{\xi-1}{\xi}\left(\frac{-b}{a}\right)\frac{R}{t}$$

$$\frac{r}{t}U(\eta) = \frac{\xi-1}{\xi}\left(\frac{2}{\alpha}\right)\frac{R}{t}$$

$r = R,\ \eta = \eta_0$

$$\frac{R}{t}U(\eta_0) = \frac{\xi-1}{\xi}\left(\frac{2}{\alpha}\right)$$

$$U(\eta_0) = \frac{\xi-1}{\xi}\left(\frac{2}{\alpha}\right)$$

Boundary Condition (5.13) is

$$\rho_1 = \rho_0\xi$$

$$r^k t^\lambda \Omega(\eta) = AR^{-\alpha}\xi$$

$r = R,\ K = -\alpha,\ \lambda = 0,\ \eta = \eta_0$

$$R^{-\alpha}t^0\Omega(\eta_0) = AR^{-\alpha}\xi$$

$$\Omega(\eta_0) = A\xi$$

$$\frac{\Omega(\eta_0)}{A} = \xi$$

$$\overline{\Omega}\left(\eta_0\right)=\xi$$

Boundary condition (5.14)

$$P_1 = \Psi \rho_0 v^2$$

$$r^{k+2}\, t^{\lambda-2} P(\eta) = \Psi\, AR^{-\alpha} \left(\frac{-b}{a}\right)^2 \frac{R^2}{t^2}$$

$$r^{k+2}\, t^{\lambda-2} P(\eta) = \Psi\, AR^{-\alpha} \frac{4}{\alpha^2} \frac{R^2}{t^2}$$

$$k = -\alpha,\ \lambda = 0,\ \eta = \eta_0$$

$$A^{2-\alpha}\, t^{-2}\, P\left(\eta_0\right) = \Psi\, AR^{-\alpha+2} \frac{4}{\alpha^2}\, t^{-2}$$

$$P\left(\eta_0\right) = \Psi\, A \frac{4}{\alpha^2}$$

$$\frac{P\left(\eta_0\right)}{A} = \Psi \frac{4}{\alpha^2}$$

$$\overline{P}\left(\eta_0\right) = \Psi \frac{4}{\alpha^2}$$

Boundary condition (5.15)

$$Pr_1 = \Psi \rho_0\, v^2$$

$$r^{k+2}\, t^{\lambda-2}\, P_r\left(\eta\right) = AR^{-\alpha} \left(\frac{-b}{a}\right)^2 \frac{R^2}{t^2}$$

$$k = -\alpha,\ \lambda = 0,\ R = r,\ \eta = \eta_0$$

$$R^{2-\alpha},\ t^{-2}\ Pr\left(\eta_0\right) = \Psi\, A \frac{4}{\alpha^2} R^{2-\alpha}\, t^{-2}$$

$$P_r\left(\eta_0\right) = \Psi A \frac{4}{\alpha^2}$$

$$\frac{P_r\left(\eta_0\right)}{A} = \Psi \frac{4}{\alpha^2}$$

$$\overline{P}_r\left(\eta\right) = \Psi \frac{4}{\alpha^2}$$

Boundary condition (5.16)

$$h_{\theta_1} = h_{\theta_0}\ \xi$$

$$r^{\frac{k+2}{2}}\ t^{(\lambda-2)/2}\ N_\theta(\eta) = \frac{\sqrt{\rho_0}\ V}{M_A}\,.\ \xi$$

$$r^{\frac{k+2}{2}}\ t^{\frac{\lambda-2}{2}}\ N_\theta(\eta) = \sqrt{\rho_0}\ \ \frac{-b}{a}\ \frac{R}{t}\ M_A\ \xi$$

$$r^{\frac{k+2}{2}}\ t^{\frac{\lambda-2}{2}}\ N_\theta(\eta) = AR^{-\alpha/2}\ \frac{2}{\alpha}\ \frac{R}{t}\ M_A\ \xi$$

$$k = -\alpha, \lambda = 0,\ r = R,\ \eta = \eta_0$$

$$R\ \frac{(2-\alpha)}{2}\ t^{-1}\ N_\theta\ (\eta) = \frac{2}{\alpha}\ \xi\ M_A^{-1}\ R\ \frac{2-\alpha}{2}\ t^{-1}$$

$$N_\theta\ (\eta_0) = \frac{2}{\alpha}\ \xi\ M_A^{-1}$$

Boundary Condition (5.17)

$$h_{z_1} = h_{z0}\ \xi$$

$$r^{\frac{k+2}{2}}\ t^{\frac{\lambda-2}{2}}\ N_z(\eta) = \sqrt{\rho_0}\ \ \frac{-b}{a}\ \frac{R}{t}\ \frac{\xi}{M_A}$$

$$r^{\frac{k+2}{2}}\ t^{\frac{\lambda-2}{2}}\ N_z(\eta) = AR^{-\alpha/2} - \frac{b}{a}\ \frac{R}{t}\ \frac{\xi}{M_A}$$

$$r = R,\ k = -\alpha,\ \lambda = 0,\ \eta = \eta_0$$

$$R^{\frac{2-\alpha}{2}}\ \ t^{-1}\ N_z\ (\eta) = A\ R^{\frac{(2-\alpha)}{2}}\ \ t^{-1}\ \frac{2}{\alpha}\ M_A^{-1}\ \xi$$

$$N_z\ (\eta) = \frac{2}{\alpha}\ M_A^{-1}\ \xi$$

Boundary condition (5.18)

$$m_1 = m_0$$

$$r^{k+3}\ t^\lambda\ W\ (\eta) = \frac{4\pi A}{(3-\alpha)}\ R^{3-\alpha}$$

$$R = r,\ k = -\alpha,\ \eta = \eta_0,\ \lambda = 0$$

$$R^{3-\alpha}\ t^0\ W\ (\eta_0) = \frac{4\pi A}{(3-\alpha)}\ R^{3-\alpha}$$

$$\frac{W(\eta_0)}{A} = \frac{4\pi}{(3-\alpha)}$$

$$\overline{W}(\eta_0) = \frac{4\pi}{(3-\alpha)}$$

**RESULT AND DISCUSSION**

Similarity solution of the problem of propagation of a cylindrical magnetogasdynamic shock wave have been obtained. For numerical calculations the flow and field variables have been taken in following non dimensional form.

$$\frac{u}{u_1} = \left(\frac{\eta}{\eta_0}\right)^{1/a} \frac{u(\eta)}{u(\eta_0)}$$

$$\frac{\rho}{\rho_1} = \left(\frac{\eta}{\eta_0}\right)^{-\alpha/a} \frac{\Omega(\eta)}{\Omega(\eta_0)}$$

$$\frac{p}{p_1} = \left(\frac{\eta}{\eta_0}\right)^{(2-\alpha)/2a} \frac{P(\eta)}{P_r(\eta_0)}$$

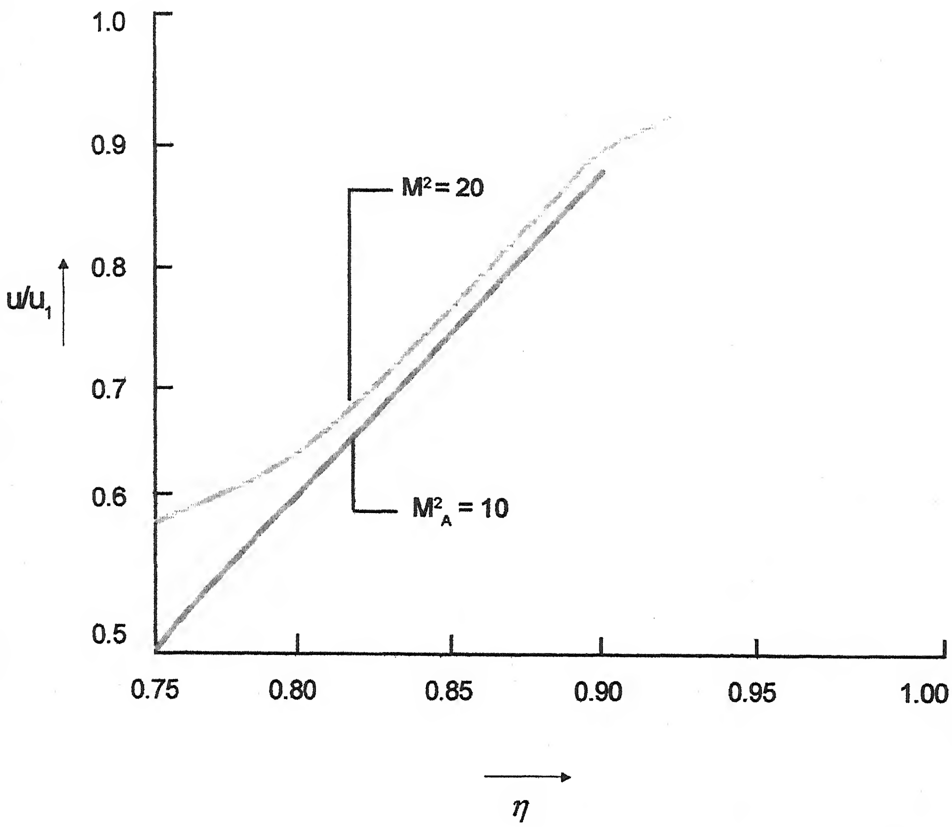
$$\frac{p}{p_{r_1}} = \left(\frac{\eta}{\eta_0}\right)^{(2-\alpha)/2a} \frac{P_r(\eta)}{P_r(\eta_0)}$$

$$\frac{h_\theta}{h_{\theta_1}} = \left(\frac{\eta}{\eta_0}\right)^{(2-\alpha)/2a} \frac{N_\theta(\eta)}{N_\theta(\eta_0)}$$

$$\frac{h_z}{h_{z_1}} = \left(\frac{\eta}{\eta_0}\right)^{(2-\alpha)/2a} \frac{N_z(\eta)}{N_z(\eta_0)}$$

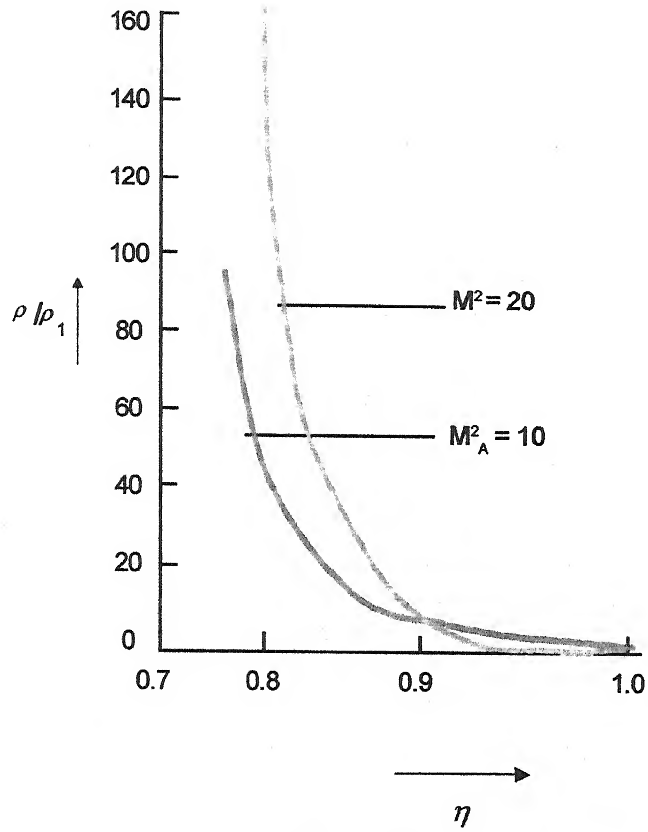
$$\frac{m}{m_1} = \left(\frac{\eta}{\eta_0}\right)^{(3-\alpha)/a} \frac{W(\eta)}{W(\eta_0)}$$

The differential equation 3.04 to 4.0 are numerically integrated by well known software matlab and computer analysis made for flow variables through graphs for certain choice of parameter $\gamma = 1.4$ $M^2 = 20$, $M_A^2 = 10$ & $\alpha = 0.25$. It is found that variation in velocity, pressure density are minimum of shock front but increase towards the line of explosions. But variable magnetic field of mass variable is maximum in shock front but decrease toward line of explosion.



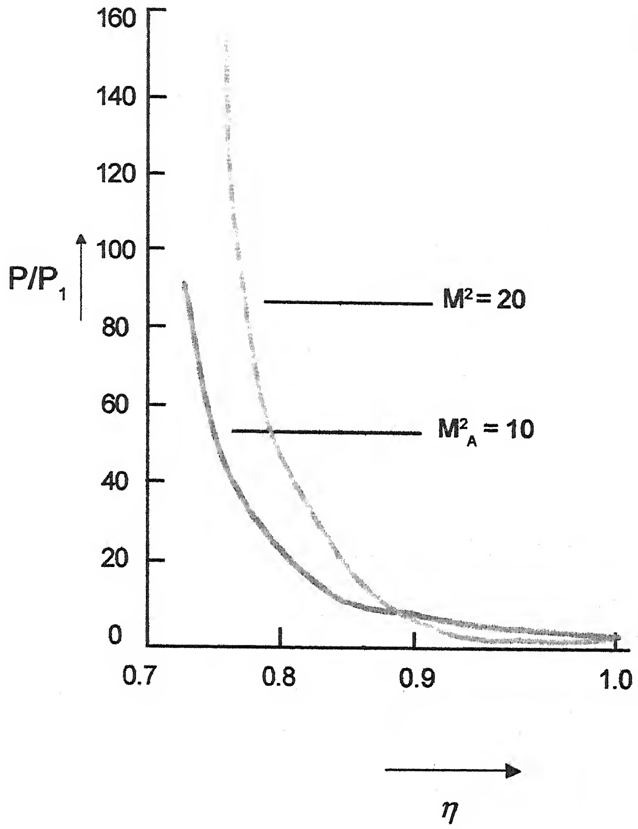

Velocity distribution

Fig. 5.1



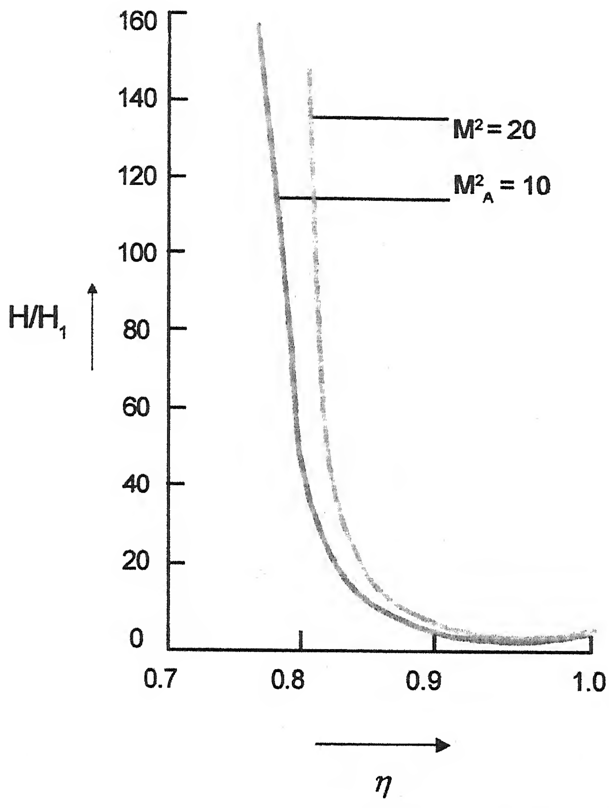

Density distribution

Fig. 5.2



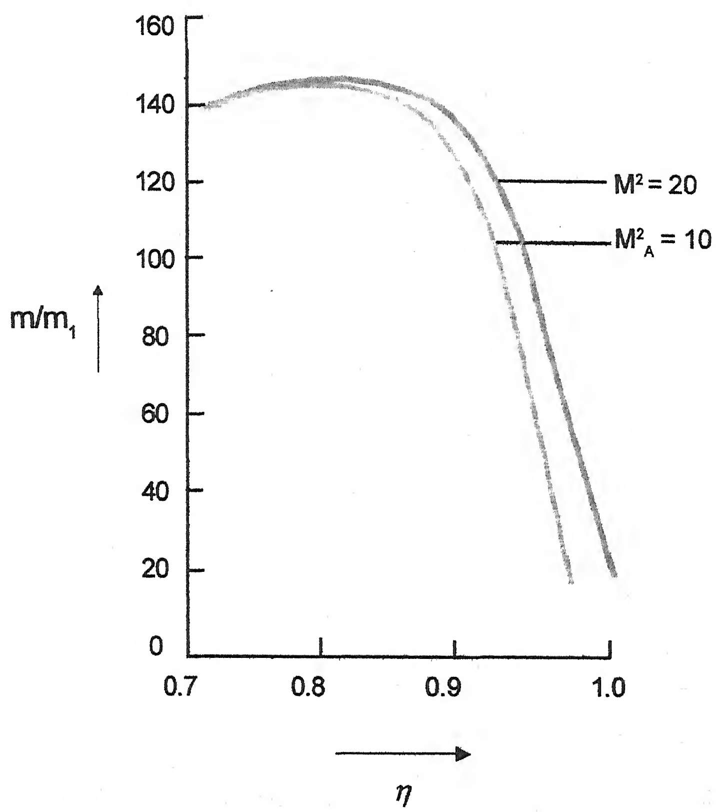

Pressure distribution

Fig. 5.3




Magnetic field

Fig. 5.4




Mass distribution

Fig. 5.5

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# Chapter – VI

# *Analytical solution of spherical shock wave in a rotating gas with axial component of magnetic field*

## INTRODUCTION

The propagation of hydromagnetic shock through a rotating gas being close to the actual situation of thunder. Kumar and Prakash [1,2,3] recently using C.C.W. [4,5,6] method have investigated the propagation of weak and strong diverging cylindrical hydromagnetic gas taking desntiy distribution variable but axial component of magnetic field of constant strength.

In this chapter, using the C.C.W. method we have considered the problems of the propagation of a converging and diverging spherical shock wave through a rotating gas under the influence of a magnetic field of constant axial, simultaneously for both weak and strong waves we have assumed initial distribution is constant. Sharma V.D. [7] & et al. have studied the problem of strong shocks in an ideal gas [7], [8], Pandey [9] use an approach based on interaction of an characteris tic shock with a weak discontinuity in a non ideal gas. T. Raja [10] also find similarity solution, for waves.

The analytical expression for shock velocity and shock strength have been obtained for weak shock under the two conditions namely when the magnetic field is weak and when the magnetic field is strong. For strong shock also we have considered two cases, i.e. when the magnetic field is strong and when $\rho_0 / p_1$ is approximately equal to $\gamma + 1/\gamma - 1$ which is purely a non magnetic case.

## BASIC EQUATIONS BOUNDARY CONDITION AND AN ANALYTIC EXPRESSION FOR SHOCK VELOCITY

The equation governing the spherical symmetrical flow of rotating gas under the influence of axial component of magnetic fields are written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{h}{\rho}\frac{\partial p}{\partial r} - \frac{v^2}{r} + \frac{\mu}{2\rho}\frac{\partial}{\partial r}H_z^2 = 0 \qquad (6.01)$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial \rho}{\partial r}\right)(vr) = 0 \qquad (6.02)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (6.03)$$

$$\frac{\partial H_Z}{\partial t} + \frac{\partial H_Z}{\partial r} + H_Z\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0 \qquad (6.04)$$

where u, p and $\rho$ are the velocity, pressure and density behind the shock front respectively while, $H_z$ is axial components of field H.

The magnetogasdynamics shock conditions can be written in terms of a single parameter.

$N = p_1/\rho_0$ as

$$\rho_1 = \rho_0 N, \quad H_1 = H_0 N, \quad U_1 = \frac{N-1}{N}U$$

$$U^2 = \frac{2N}{(\gamma+1)-(\gamma-1)N}\left[a_0^2 + \frac{b_0^2}{2} + \{(2-\gamma)N+\gamma\}\right]$$

$$p_1 = p_0 \frac{2\rho_0(N-1)}{(\gamma+1)-(\gamma-1)N}\left[a_0^2 + \frac{\gamma-1}{4}b_0^2(N-1)^2\right] \qquad (6.05)$$

where 0 and 1 denote respectively the states immediately ahead and behind the shock front , U the shock velocity, a the sound speed $(\gamma\, p_0/\rho_0)$ and the Alfven speed is $(\mu H_0^2/\rho_0)$

where

$H_0^2 = Hz_0^2$

In the equilibrium state we have

$$\frac{1}{\rho_0}\frac{d\rho_0}{dr} - \frac{V^2}{r} = 0$$

Integrating the proceeding equation we get.

$$P = K_1 \log \frac{A}{r} \text{ and } a_0 = K_2 \left[\log \frac{A}{r}\right]^{1/2}$$

where $K_1 = \rho' V^2$

$$K_2 = \left(\frac{\gamma K_1}{\rho'}\right)^{1/2} \text{ and}$$

$\rho' V^2 \log A$ is constant of integration.

## WEAK SHOCKS

For a very weak shock we take the parameter N as

$$\frac{\rho}{\rho_0} = N = 1 + \varepsilon, \qquad \varepsilon << 1$$

Now consider the two cases of weak and strong magnetic fields.

**Case – I**

When the magnetic field is weak i.e.

$$b_0^2 << a_0^2 \left(\text{i.e. } \mu H_0^2 << \gamma \rho_0\right)$$

and therefore, the boundary conditions (5) reduce to

$$\rho_1 = \rho_0 (1+\varepsilon)$$

$$u_1 = b_0$$

$Hz_1 = Hz_0\ (1+\varepsilon)$

$Pl = p_0\ (1+\gamma)$

$$U = \left(1 + \frac{\gamma+1}{4}\right) a_0 \qquad (6.06)$$

**Case 2**

When magnetic field is strong $b_0^2 >> a_0^2$ under these circumstances the boundary condition (6.05) reduce to

$\rho_1 = \rho_0\ (1+\varepsilon)$

$u_1 = b_0$

$Hz_1 = Hz_0\ (1+\varepsilon)$

$p_1 = p_0\ (1+\varepsilon)$

$$U = \left(1 + \frac{3}{4}\varepsilon\right) b_0 \qquad (6.07)$$

**STRONG SHOCK**

**Case – I**

When the magnetic field is strong $b_0^2 >> a_0^2$ $\left(\text{i.e. } \mu H_0^2 >> \gamma p_0\right)$ under this condition the boundary condition (6.05) becomes.

$p_1 = p_0\ N$

$H_1 = H_0\ N$

$$U_1 = \frac{N-1}{N}\ U$$

$$\frac{p_1}{p_0} = \frac{XU^2}{a_0^2} + L$$

where

$$X \quad = \quad \frac{\gamma\,(\gamma+1)(N-1)^3}{2N\,\{(2-\gamma)\,N+\gamma\}} \text{ and}$$

$$L \quad = \quad \frac{(\gamma+1)\,N-(\gamma-1)}{(\gamma+1)-(\gamma-1)N} \text{ and} \tag{6.08}$$

**Case –II**

For strong shock when $b_0^2 << a_0^2$ (i.e. when $N \to [(\gamma+1)/(\gamma-1)]$) is small the field may be regarded as independent of magnetic field.

For converging shock the characteristics form of system of equation (6.02) is

$$dp + \mu\,(H_z\,dH_z) + pc\,du + \left[\frac{\rho c^2 u}{u-c} + \frac{\rho c v^2}{u-c}\right]\frac{dr}{r} = 0 \tag{6.09}$$

where as the characteristic form of diverging shock is

$$dp + \mu\,(H_z\,dH_z) + pc\,du + \left[\frac{\rho c^2 u}{u+c} - \frac{\rho c v^2}{u+c}\right]\frac{dr}{r} = 0\,d \tag{6.10}$$

where $c^2 = a^2 + b^3 = \dfrac{\gamma p}{\rho} + \dfrac{\mu H_z^2}{\rho}$

and $\quad H_0^2 = H_z^2$

**WEAK SHOCK WITH WEAK MAGNETIC FIELD**

Substituting shock condition (6.06) into (6.09) and neglecting the second and higher order term of ε since ε << we get.

$$\left\{\frac{\mu}{\gamma p_0}\,(H_{z0}^2)\right\} d + \left\{\frac{dp_0}{p_0} - \frac{da_0}{a_0} - \frac{dr}{r} - \frac{2dp_0}{\gamma p_0}\right\} = 0 \tag{6.11}$$

Now substituting

$$\frac{dp_0}{p_0}, \quad \frac{da_0}{a_0}, \text{ and } \frac{\mu Hz_0^2}{\gamma p_0}$$

in equation (6.11) we get

$$\frac{d\varepsilon}{\varepsilon} = \left[\frac{[\gamma-4]}{\gamma\beta_1^2} + \frac{1}{\beta_1^2}\log\frac{A}{r}\right]\frac{dr}{r} \tag{6.12}$$

where $\beta_1^2 = \dfrac{\mu H_{z0}^2}{\gamma k_1}$

$\beta_1^2 = 0$

Integrating of equation (6.12) yields

$$\varepsilon = \overline{K}_3\, r^{[\gamma-4]/\gamma\beta_1^2} \exp\left\{-\frac{1}{2\beta_1^2}\left(\log\frac{A}{r}\right)^2\right\} \tag{6.13a}$$

where log to constant of integration in case of diverging wave shock with weak magnetic field, we have the relation.

$$\varepsilon = K_3\, r^{-1/2}\left(\log\frac{A}{r}\right)^{-q_1} \exp\left\{-\frac{3}{8}\beta_1^2 (\log)\frac{A}{r})^{-1}\right\} \tag{6.13b}$$

where log $K_3$ is constant of integration and

$$q_1 = \left(\frac{3}{4} + \frac{\beta_1^2}{4}\right)$$

Hence from equation (6.13a) and (6.13b) we have

$$\frac{u}{a_0} = 1 + \frac{\gamma+1}{4}\overline{K}_3\, r^{\left[\gamma-\left(1-4\beta_1^2\right)-4\right]/\gamma\beta_1^2} \exp\left\{-\frac{1}{2\beta_1^2}\left(\log\frac{A}{r}\right)^2\right\} \tag{6.14}$$

$$\frac{u}{a_0} = 1 + \frac{\gamma+1}{4}\overline{K}_3\, r^{-1/2}\left(\log\frac{A}{r}\right)^{-q_1} \exp\left\{-\frac{3}{8}\beta_1^2\left(\log\frac{A}{r}\right)^{1}\right\} \tag{6.15}$$

for converging and diverging shock respectively.

**WEAK SHOCK WITH STRONG MAGNETIC FIELD**

Substituting the shock conditions (6.07) in (6.09) we get

$$d\varepsilon + \left\{ \frac{dp_0}{p_0} - \frac{\mu H_0^2}{\gamma p_0} \left( \frac{dr}{r} + \frac{db_0}{b_0} \right) - \frac{2dp_0}{\gamma p_0} \right\} = 0 \tag{6.16}$$

Substituting the respective quantities in equation (6.16) we get

$$\frac{d\varepsilon}{\varepsilon} = \left[ \frac{\gamma\left(1+\beta_1^2-2\right)}{\gamma} \right] \left( \log \frac{A}{r} \right)^{-1} \frac{dr}{r} \tag{6.17}$$

integrating equation (6.17) we get

$$\varepsilon = K_4 \left( \log \frac{A}{r} \right)^{-q_2} \tag{6.18a}$$

where log $\overline{K}_4$ is constant of integration and

$$q_2 = \frac{\left[\gamma\left(1+\beta_1^2\right)-2\right]}{\gamma}$$

when we consider diverging shock with strong magnetic field we get

$$\varepsilon = K_4 \log \left( \frac{A}{r} \right)^{\beta_1^2} - 1 \exp \left\{ -2\beta_1^2 \left(1-\beta_1^2\right) \left( \log \frac{A}{r} \right)^{-1} \right\} \tag{6.18b}$$

where log $\overline{K}_4$ is constant of integration equation (6.18a) and (6.18b) disclose that

$$\frac{u}{b_0} = 1 + \frac{3}{4}\overline{K}_4 \left( \log \frac{A}{r} \right)^{-q_2} \tag{6.19}$$

$$\frac{u}{a_0} = 1 + \frac{3}{4} K_4 \left( \log \frac{A}{r} \right)^{\beta_2^2} - 1 \exp \left\{ -2\beta_1^2 \left(1-\beta_1^2\right) \left( \log \frac{A}{r} \right)^{-1} \right\} \tag{6.20}$$

and diverging shocks respectively.

**STRONG SHOCK**

Substituting shock condition (6.08) into (6.09) we get

$$dU^2\left[\frac{x}{y}-\frac{N-1}{2}\left(\frac{X}{N}\right)^{1/2}+U^2\left[\frac{X}{Y}\,\frac{dp_0}{p_0}-\frac{2x}{y}\,\frac{da_0}{a_0}+\frac{x\,(N-1)}{(N-1)-\sqrt{Nx}}\,\frac{dr}{r}\right]\right]$$

$$+\left[L\,\frac{dp_0}{p_0}-\frac{\gamma k_1\,N^2\,\beta_1^2}{p_0}\left\{\frac{(N-1)+\sqrt{NX}}{(N-1)-\sqrt{Nx}}\right\}\frac{dr}{r}+\frac{N\sqrt{Nx}}{[(N-1)-\sqrt{Nx}]}\right]=0 \tag{6.21}$$

Substituting the respective values we get.

$$\frac{du^2}{dr}+\frac{B}{r}U^2-cr^{-1}=0 \tag{6.22}$$

where B = B'/M

C = C'/M

$$B'=\frac{X(N-1)}{(N-1)\sqrt{XN}}$$

$$M=\frac{x}{y}-\frac{(N-1)}{2}\left(\frac{x}{N}\right)^{1/2}$$

and

$$C'=\frac{Lk_1}{\rho'}+\frac{\gamma k_1 N^2\beta_1^2}{\rho'}\left\{\frac{(N-1)+\sqrt{NX}}{(N-1)-\sqrt{NX}}\right\}-\frac{N\sqrt{NX}}{(N-1)-\sqrt{NX}}$$

Integration of (6.22) yields

$$U^2=K_5r^{-\beta}+\frac{C}{B} \tag{6.23}$$

where $\overline{K}_5$ is constant of integration. In the case of diverging strong shock with strong magnetic field we have

$$U_2 = K_5 r^{-\beta} + \frac{C}{B} \tag{6.24}$$

Where $K_5$ is constant of integration, finally for converging and diverging shock we have

$$\frac{U^2}{a_0^2} = \frac{1}{K_2^2 \log \frac{A}{r}} \left( \overline{K}_5 \, r^{-\beta} + \frac{C}{B} \right) \tag{6.25}$$

and

$$\frac{U^2}{a_0^2} = \frac{1}{K_2^2 \log \frac{A}{r}} \left( \overline{K}_5 \, r^{-\beta} + \frac{C}{B} \right) \tag{6.26}$$

respectively

In the last, the expressions for the velocity, the density, and the pressure of the gas just behind the shock surface are

## STRONG SHOCK WITH WEAK MAGNETIC FIELD

$$U = K_2 \overline{K}_3 \left( \log \frac{A}{r} \right)^{1/2} r^{(\gamma-4)/\gamma\beta_1^2} \exp \left\{ -\frac{1}{2\beta_1^2} \log \left( \frac{A}{r} \right)^2 \right\} \tag{6.27a}$$

$$U = K_2 \overline{K}_3^{-1/2} \left( \log \frac{A}{r} \right)^{(1/2q_1)} \exp \left\{ -\frac{3}{8} \beta_1^2 \log \left( \frac{A}{r} \right)^{-1} \right\} \tag{6.27b}$$

$$\rho = \rho' \left[ 1 + \overline{K}_3 \, r^{(\gamma-4)/\gamma\beta_1^2} \exp \left\{ -\frac{1}{2\beta_1^2} \left( \log \frac{A}{2} \right)^2 \right\} \right] \tag{6.28a}$$

$$\rho = \rho' \left[ 1 + K_3 \, r^{-1/2} \left( \log \frac{A}{r} \right)^{-q_1} \exp \left\{ -\frac{3}{8} \beta_1^2 \left( \log \frac{A}{r} \right)^{-1} \right\} \right] \tag{6.28b}$$

$$p = k_1 \left( \log \frac{A}{r} + \gamma k_3 \, r^{(\gamma-4)/\gamma\beta_1^2} \left( \log \frac{A}{r} \right) \exp \left\{ -\frac{1}{2\beta_1^2} \left( \log \frac{A}{r} \right)^2 \right\} \right) \tag{6.29a}$$

$$p = k_1 \left( \log \frac{A}{r} + \gamma k_3 r^{-1/2} \left( \log \frac{A}{r} \right)^{1-q_1} \exp \left\{ -\frac{3}{8} \beta_1^2 \left( \log \frac{A}{r} \right)^{-1} \right\} \right) \quad (6.29b)$$

**WEAK SHOCK WAVE WITH STRONG MAGNETIC FIELD**

$$U = \beta_1 K_2 \overline{K}_4 \left( \log \frac{A}{r} \right)^{-q_2} \quad (6.30a)$$

$$U = \beta_1 K_2 \overline{K}_4 \left( \log \frac{A}{r} \right)^{\beta_1^2} - 1 \exp \left\{ -2\beta_1^2 \left(1 - \beta_1^2\right) \left( \log \frac{A}{r} \right)^{-1} \right\} \quad (6.30b)$$

$$\rho = \rho' \left[ 1 - \overline{K}_4 \left( \log \frac{A}{r} \right)^{-q_2} \right] \quad (6.31a)$$

$$\rho = \rho' \left[ 1 + K_4 \left( \log \frac{A}{r} \right)^{-\beta_1^2} - 1 \ \exp \left\{ -2\beta_1^2 \left(1 - \beta_1^2\right) \log \left( \frac{A}{R} \right)^{-1} \right\} \right] \quad (6.31b)$$

$$P = K_1 \left[ \log \frac{A}{r} \ \gamma \overline{K}_4 \left( \log \frac{A}{r} \right)^{1-q_4} \right] \quad (6.32a)$$

$$p = k_1 \left[ \log \frac{A}{r} + \gamma \overline{K}_4 \left( \log \frac{A}{r} \right)^{\beta_1^2} \right] \exp \left\{ -2\beta_1^2 \left(1 - \beta_1^2\right) \log \left( \frac{A}{r} \right)^{-1} \right\} \quad (6.32b)$$

**STRONG SHOCK WITH STRONG MAGNETIC FIELD**

$$U = \frac{(N-1)}{N} \left[ \overline{K}_5 \ r^{-\beta} + \frac{C}{B} \right]^{1/2} \quad (6.33a)$$

$$U = \frac{(N-1)}{N} \left[ \overline{K}_5 \ r^{-\beta} + \frac{C}{B} \right]^{1/2} \quad (6.33b)$$

$\rho = \rho' N$ for converging diverging cases

$$p = K_1 \log \frac{A}{\gamma} \left[ \frac{X\left(\overline{K}_5\, r^{-\beta} + C/B\right)}{K_2 \left(\log \dfrac{A}{r}\right)^{1/2}} + L \right] \tag{6.34}$$

$$p = K_1 \log \frac{A}{\gamma} \left[ \frac{X\left(K_5\, r^{-\beta} + C/B\right)}{K_2 \left(\log \dfrac{A}{r}\right)^{1/2}} + L \right] \tag{6.35b}$$

where section (a) and (b) denote the expression for converging and diverging shocks respectively.

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# Chapter-VII

# *Self Similar Model of Radiative Shock Waves in Magnetohydrodynamics with Magnetic Effect*

## INTRODUCTION

In this chapter we consider propagation of shock wave under the effect of radiation taking magnetic effect in to account of an instantanous release of energy in a non-uniform equilibrium conditions. The disturbance are headed by a shock of variable strength. This model is of considerable physical interest in sonic booms.

The medium ahead of the shock is assumed to be inhomogeneous and at rest the shock position in this problem is given by

The theory of shock waves and related flows has attracted a renewed interest in connection with the phenomena associated with laser production of plasma, nuclear detonation and astrophysical situations. Ray [1] obtained an analytic solution in the case of a central explosion in a gravitating mass of equilibrium in which the disturbance was headed by a shock surface of variable strength. Ojha [2] considered the propagation of explosion waves in a steller model in the shock strength does not remain constant in general and mach number of the shock is a function of time. Ojha and onkar [3] studied the propagation of shock in inhomogeneous self gravitating gaseous mass are headed. Tayar [4], Carrusetal [5], Klynch [6], Sedov [7] have accumulated and extensive to literature on self similar model of phenomena for the propagation of shock wave in gas dynamics and in the formation of stars and super nova explosions. Summers and whitham [8] have discussed qualitative analysis of self similar solution to the problem of unsteady spherical symmetric motion of self gravitating gas Sakurai [9] has considered the problem of shock wave arriving at the edge of a gas in a medium in which density varying as power law. The flare generating shock waves has been studied by Hundhanses [10]. Deb Ray and Bhownick [11] have obtained the self similar solution for central explosion in stars with radiation flux when the shock is isothermal & transparent.

$$R = A\, t^{\mu} \tag{7.01}$$

where $\mu$ and A are constant & $\mu < 1$

We assume that the density disturbance is given by

$$\rho_0 = br^{\beta} \tag{7.02}$$

where b & $\beta$ are constants.

The magnetic field disturbance the magnetic field distribution is taken cc & Rosenate & Frankenthal [12] as

$$h_0 = cr^{\delta} \tag{7.03}$$

directed tangentially to the advancing shock front where c & $\delta$ are constants.

## EQUATIONS OF MOTION

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} = 0 \tag{7.04}$$

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial r} - \frac{\gamma P}{\rho}\left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r}\right) = 0 \tag{7.05}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \left(\frac{h_\theta + h_z}{\rho r}\right)\frac{\partial}{\partial r}\left(h_\theta\, r + h_z\, r\right) = 0 \tag{7.06}$$

$$\frac{\partial h_\theta}{\partial t} + u \frac{\partial h_\theta}{\partial r} + h_\theta \frac{\partial u}{\partial r} = 0 \tag{7.07}$$

$$\frac{\partial h_z}{\partial t} + u \frac{\partial h_z}{\partial r} + h_z \frac{\partial u}{\partial r} = 0 \tag{7.08}$$

Where $\rho, u, p, h_\theta, h_z, \gamma, m$ are the density, velocity, pressure, azumethal magnetic field, transverse magnetic field and ratio of specific heats of the gas, mass respectively.

We introduce the similarity variable $\eta = \frac{r}{R(t)}$ and take the solution of fundamental equation in the form

$$u = \dot{R}\, V(\eta) \tag{7.09}$$

$$\rho = \rho_0\, D(\eta) \tag{7.10}$$

$$p = \rho_0\, \dot{R}^2\, P(\eta) \tag{7.11}$$

$$h_\theta = \sqrt{\rho_0}\ \dot{R}\ H_0\ (\eta) \tag{7.12}$$

$$h_z = \sqrt{\rho_0}\ \dot{R}\ H_z\ (\eta) \tag{7.13}$$

where $V, D, P, H_\theta, H_z$ are function of $\eta$ only and $\dot{R} = \dfrac{dR}{dt}$ the shock velocity.

The jump condition for a shock wave (cf summer & Whitworth [13]) are

$$u_1 = \frac{2\dot{R}}{(\gamma+1)} \tag{7.14}$$

$$P_1 = \frac{2\rho_0 \dot{R}^2}{(\gamma+1)} \tag{7.15}$$

$$\rho_1 = \frac{(\gamma+1)\rho_0}{(\gamma-1)} \tag{7.16}$$

$$h_{\theta_1} = \frac{(\gamma+1)}{(\gamma-1)} h_{\theta_0} \tag{7.17}$$

$$h_{z_1} = \frac{(\gamma+1)}{(\gamma-1)}\ h_{z_0} \tag{7.18}$$

where suffix 1 denotes the volumes of flow variables immediately behind the shock front.

## SOLUTION OF EQUATION OF MOTION

By Equation (7.01)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{\rho \partial u}{\partial r} = 0 \tag{7.01}$$

By condition (7.09)

$$u = \dot{R}\ V(\eta)$$

$$\frac{\partial u}{\partial r} = \dot{R}\, V'\ \frac{\partial \eta}{\partial r}$$

$$\because \qquad \eta = \frac{r}{R(t)}$$

$$\frac{\partial \eta}{\partial r} = \frac{1}{R}$$

$$\frac{\partial \eta}{\partial r} = \frac{\eta}{r}$$

$$\frac{\partial u}{\partial r} = \dot{R} V' \frac{\eta}{r}$$

$$\eta = \frac{r}{R(t)}$$

$$\frac{\partial \eta}{\partial t} = \frac{-\eta}{R} \dot{R}$$

By condition (7.10)

$$\frac{\partial \rho}{\partial t} = \rho_0 D' \frac{\partial \eta}{\partial t}$$

$$\frac{\partial \rho}{\partial t} = -\rho_0 D' \frac{\eta}{R} \dot{R}$$

By condition (7.10)

$$\rho = \rho_0 D(\eta)$$

$$\frac{\partial \rho}{\partial r} = \rho_0 D' \frac{\partial \eta}{\partial r} + D \frac{\partial \rho_0}{\partial r}$$

$$\frac{\partial \rho}{\partial r} = \frac{\rho_0}{R} \left[ D' + \frac{D\beta}{\eta} \right]$$

By condition (7.02)

$$\rho_0 = br^{\beta}$$

$$\frac{\partial \rho_0}{\partial r} = \frac{\beta \left( br^{\beta} \right)}{r}$$

$$\frac{\partial \rho_0}{\partial r} = \frac{\beta \rho_0}{r}$$

$$\frac{\partial \rho_0}{\partial r} = \frac{\beta \rho_0 \eta}{R}$$

substituting these values in equation (7.01)

$$-\rho_0 D^1 \frac{\eta}{R} \dot{R} + V \dot{R} \frac{\rho_0}{R} \left[ D' + \frac{D\beta}{\eta} \right] + \rho_0 \dot{R} V' \frac{\eta}{r} = 0$$

$$-\rho_0 \frac{D^1\eta}{R} + \frac{V}{R}\left[D' + \frac{D\beta}{\eta}\right] + \frac{DV'\eta}{\eta R} = 0$$

$$-D'\eta + V\left[D' + \frac{D\beta}{\eta}\right] + DV' = 0$$

$$(-\eta + V)D' + \left[\frac{V\beta}{\eta} + V'\right]D = 0 \tag{7.19}$$

By equation (7.05)

$$\frac{\partial p}{\partial t} + u\frac{\partial P}{\partial r} - \frac{\gamma P}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0$$

By Condition (7.11)

$$P = \rho_0 \dot{R}^2 P(\eta)$$

$$\frac{\partial \rho}{\partial r} = \frac{\rho_0}{R}\left[D' + \frac{D\beta}{\eta}\right]$$

substituting these values in equation (7.05), we get

$$\frac{\dot{R}^3\rho_0}{R}\left[-P'\eta + 2P\frac{(\mu-1)}{\mu}\right] + \frac{\dot{R}V\rho_0\dot{R}^2}{R}\left[-P' + \frac{P\beta}{\eta}\right] - \frac{\gamma\rho_0\dot{R}^2P}{\rho_0 0} - \frac{\rho_0 D'\eta\dot{R}}{R} + \frac{V\dot{R}\rho_0}{R}\left[D' + \frac{D\beta}{\eta}\right] = 0$$

$$(V-\eta)P^1 + 2P\frac{(\mu-1)}{\mu} + \left[\frac{\beta V}{\eta} + \frac{2(\mu-1)}{\mu}\right]P - \frac{\gamma P}{D}\left[(V-\eta)D' + \frac{\beta DV}{\eta}\right] = 0 \tag{7.20}$$

Now, taking equation (7.06)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{(h_\theta + h_z)}{\rho r}\frac{\partial}{\partial r}(h_\theta . r + h_z . r) = 0$$

By condition (7.09)

$$u = \dot{R}V(\eta)$$

$$\frac{\partial u}{\partial r} = \frac{\dot{R}V'}{R}$$

$$\frac{\partial P}{\partial t} = \sqrt{\rho_0}\left[P'\frac{\partial \eta}{\partial t}.\dot{R}^2 + 2\dot{R}\ddot{R}P\right]$$

$$= \sqrt{\rho_0}\left[P'\left(\frac{-\eta\dot{R}}{R}\right)\dot{R}^2 + 2\dot{R}\ddot{R}P\right]$$

By condition (7.01)

$$R = A t^{\mu}$$

$$\dot{R} = \mu\, A t^{\mu-1}$$

$$\ddot{R} = \mu\,(\mu-1)\, A t^{\mu-2}$$

$$\ddot{R} = \frac{(\mu-1)}{\mu}\dot{R}^{2}$$

Then

$$\frac{\partial P}{\partial t} = \sqrt{\rho_0}\left[-\frac{P'\eta\dot{R}}{R}\dot{R}^{2} + 2\dot{R}\frac{(\mu-1)}{\mu}\dot{R}^{2}P\right]$$

$$\frac{\partial P}{\partial t} = \frac{\dot{R}^{3}\rho_0}{R}\left[-P'\eta + 2P\frac{(\mu-1)}{\mu}\right]$$

again, by condition (7.11)

$$P = \rho_0\,\dot{R}^{2}\,P$$

$$\frac{\partial P}{\partial r} = \dot{R}^{2}\left[P\frac{\partial\rho_0}{\partial r} + P'\frac{\partial\eta}{\partial r}\rho_0\right]$$

$$\frac{\partial P}{\partial r} = \frac{\dot{R}^{2}\rho_0}{R}\left[\frac{P\beta}{\eta} + P'\right]$$

$$\frac{\partial\rho}{\partial t} = \rho_0\,D'\,\frac{\eta}{R}\,\dot{R}$$

$$\frac{\partial u}{\partial t} = \left[\dot{R}V'\frac{\partial\eta}{\partial t} + V\frac{\partial\dot{R}}{\partial t}\right]$$

$$\frac{\partial u}{\partial t} = \left[\dot{R}\,V'\left(-\frac{\eta}{R}\dot{R}\right) + V\frac{(\mu-1)}{\mu}\dot{R}^{2}\right]$$

$$\frac{\partial u}{\partial t} = \frac{\dot{R}^{2}}{R}\left[-\eta\,V' + V\frac{(\mu-1)}{\mu}\right]$$

By condition (7.12)

$$h_{\theta} = \sqrt{\rho_0}\;\dot{R}\,H_{\theta}(\eta)$$

$$\frac{\partial h_\theta}{\partial r} = \dot{R}\left[H_\theta' \frac{\partial \eta}{\partial r}\sqrt{\rho_0} + H_\theta \frac{\partial \rho_0^{1/2}}{\partial r}\right]$$

$$= \dot{R}\left[H_\theta' \frac{\sqrt{\rho_0}}{R} + \frac{H_\theta \beta}{2\eta}\sqrt{\rho_0}\right]$$

$$\frac{\partial h_\theta}{\partial r} = \frac{\dot{R}\sqrt{\rho_0}}{R}\left(H_\theta' + \frac{H_\theta \beta}{2\eta}\right)$$

again, by condition (7.13)

$$h_z = \sqrt{\rho_0}\ \dot{R}\ H_z(\eta)$$

$$\frac{\partial h_z}{\partial r} = \dot{R}\left[\sqrt{\rho_0}\ H_z' \frac{\partial \eta}{\partial r} + H_z \frac{\partial}{\partial r}\rho_0^{1/2}\right]$$

$$\frac{\partial h_z}{\partial r} = \frac{\dot{R}\sqrt{\rho_0}}{R}\left[H_z' + \frac{H_z \beta}{2\eta}\right]$$

By condition (7.11)

$$P = \rho_0\ \dot{R}^2\ P(\eta)$$

$$\frac{\partial P}{\partial r} = \frac{\rho_0\ \dot{R}^2}{R}\left[P' + \frac{P\beta}{\eta}\right]$$

substituting these values in this equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{(h_\theta + h_z)}{\rho}\left(\frac{\partial h_\theta}{\partial r} + \frac{\partial h_z}{\partial r}\right) + \frac{(h_\theta + h_z)^2}{\rho r} = 0$$

so, we get

$$\frac{\dot{R}^2}{R}\left[-\eta V^1 + V\frac{(\mu-1)}{\mu}\right] + \frac{\dot{R}V.\dot{R}V'}{R} + \frac{\rho_0\dot{R}^2}{\rho_0 DR}\left[P' + \frac{P\beta}{\eta}\right]$$

$$+ \frac{\sqrt{\rho_0}\ \dot{R}[H_\theta + H_z]}{\rho_0 D}\left[\frac{(H_\theta + H_z)\beta}{2\eta} + (H_\theta' + H_z')\right]\frac{\dot{R}\sqrt{\rho_0}}{R} + \frac{(H_\theta + H_z)^2}{\rho_0 D.\eta R}\rho_0\dot{R}^2 = 0$$

$$-\eta V^1 + \frac{V(\mu-1)}{\mu} + VV' + \frac{1}{D}\left[P' + \frac{P\beta}{\eta}\right] + (H_\theta + H_z)\left[(H_\theta' + H_z') + \frac{(H_\theta + H_z)}{2\eta}\beta\right] + \frac{(H_\theta + H_z)^2}{D\eta} = 0 \tag{7.21}$$

Using equation (7.07)

$$\frac{\partial h_\theta}{\partial t} + u\frac{\partial h_\theta}{\partial r} + h_\theta \frac{\partial u}{\partial r} = 0 \qquad (7.7)$$

By condition (7.12)

$$h_\theta = \sqrt{\rho_0}\ \dot{R}\ H_\theta\ (\eta)$$

$$\frac{\partial h_\theta}{\partial t} = \sqrt{\rho_0}\left[H_\theta' \frac{\partial \eta}{\partial t} \dot{R} + H_\theta \ddot{R}\right]$$

$$= \sqrt{\rho_0}\left[H_\theta' \frac{-\eta}{R} \dot{R}.\dot{R} + H_\theta \frac{(\mu-1)\dot{R}^2}{\mu R}\right]$$

$$\frac{\partial H_\theta}{\partial t} = \sqrt{\rho_0}\ \frac{\dot{R}^2}{R}\left[-\eta H_\theta' + \frac{(\mu-1)}{\mu} H_\theta\right]$$

again, $h_\theta = \sqrt{\rho_0}\ \dot{R}\ H_\theta\ (\eta)$

$$\frac{\partial h_\theta}{\partial r} = \dot{R}\left[H_\theta^1 \frac{\partial \eta}{\partial r} \sqrt{\rho_0} + H_\theta \frac{\partial \rho_0^{1/2}}{\partial r}\right]$$

$$\dot{R}\left[H_\theta' \frac{\rho_0}{R} + \frac{H\theta\beta}{2\eta R} \sqrt{\rho_0}\right]$$

$$\frac{\partial h_\theta}{\partial r} = \frac{\dot{R}\sqrt{\rho_0}}{R}\left[H_\theta' + \frac{H_\theta \beta}{2\eta}\right]$$

By condition (7.09)

$$u = \dot{R}\ V(\eta)$$

$$\frac{\partial u}{\partial r} = \dot{R}\, V' \ \frac{\partial \eta}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{\dot{R} V'}{R}$$

substituting these values in eq. (7.07) we get

$$\frac{\sqrt{\rho_0}\,\dot{R}^2}{R}\left[-\eta H_\theta' + \frac{(\mu-1)}{\mu} H_\theta + \frac{\dot{R}V\dot{R}\sqrt{\rho_0}}{R}\left[H_\theta' + \frac{H_\theta \beta}{2\eta}\right] + \sqrt{\rho_0}\ \dot{R}\, H_\theta \frac{\dot{R}V'}{R} = 0\right.$$

$$-\eta H_\theta' + \frac{(\mu-1)}{\mu} H_\theta + H_\theta' V + \frac{V H_\theta \beta}{2\eta} + V' H_\theta = 0$$

$$(V-\eta)\,H_\theta' + \left[\frac{(\mu-1)}{\mu} + \frac{V\beta}{2\eta} + V'\right] H_\theta = 0 \tag{7.22}$$

By equation (7.08)

$$\frac{\partial h_z}{\partial t} + u\,\frac{\partial h_z}{\partial r} + h_z\,\frac{\partial u}{\partial r} = 0$$

By condition (7.13)

$$h_z = \sqrt{\rho_0}\;\dot{R}\,H_z(\eta)$$

$$\frac{\partial h_z}{\partial t} = \sqrt{\rho_0}\left[H_z'\,\frac{\partial \eta}{\partial t}\,\dot{R} + H_z\,\ddot{R}\right]$$

$$= \sqrt{\rho_0}\left[H_z^1\,\frac{-\eta}{R}\,\dot{R}\,\dot{R} + H_z\,\frac{(\mu-1)\,\dot{R}^2}{\mu\,R}\right]$$

$$\frac{\partial h_z}{\partial t} = \frac{\sqrt{\rho_0}\;\dot{R}^2}{R}\left[-\eta H_z' + \frac{(\mu-1)}{\mu}\,H_z\right]$$

again $h_z = \sqrt{\rho_0}\;\dot{R}\,H_z(\eta)$

$$\frac{\partial h_z}{\partial r} = \dot{R}\left[H_z'\,\frac{\partial \eta}{\partial r}\,\sqrt{\rho_0} + H_z\,\frac{\partial}{\partial r}\,\rho_0^{1/2}\right]$$

$$= \dot{R}\left[\frac{H_z'\sqrt{\rho_0}}{R} + \frac{H_z\,\beta\sqrt{\rho_0}}{2\eta R}\right]$$

$$\frac{\partial h_z}{\partial r} = \frac{\dot{R}\sqrt{\rho_0}}{R}\left[H_z' + \frac{H_z\,\beta}{2\eta}\right]$$

By equation (7.09)

$$u = \dot{R}\,V(\eta)$$

$$\frac{\partial u}{\partial r} = \dot{R}\,V'\,\frac{\partial \eta}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{\dot{R}V'}{R}$$

Substituting these values in equation (7.08)

$$\sqrt{\rho_0}\ \frac{\dot{R}^2}{R}\left[-\eta\ H_z^{'} + \frac{(\mu-1)}{\mu}\ H_z\right] + \frac{\dot{R}V.\ \dot{R}\sqrt{\rho_0}}{R}\left[\left(H_z^{'} + \frac{H_z\beta}{2\eta}\right)\right] + \sqrt{\rho_0}\ \dot{R}\ H_z\ \frac{\dot{R}V'}{R} = 0$$

$$-\eta\ H_z^{'} + \frac{(\mu-1)}{\mu}\ H_z\ + H_z^{'}\ V + \frac{VH_z\ \beta}{2\eta} + V'\ H_z = 0$$

$$(V-\eta)\ H_z^{'}\ + \left[\frac{(\mu-1)}{\mu} + \frac{V\beta}{2\eta} + V'\right]\ H_z = 0 \tag{7.23}$$

Equation (7.19-7.22) can be reduce in new form

$$D' = \frac{1}{(V-\eta)}\left[\frac{V\beta}{\eta} + V'\right]\ D \tag{7.24}$$

$$P' = \frac{1}{(V-\eta)}\left[\left\{\frac{\beta V}{\eta} + \frac{2(\mu-1)}{\mu}\right\}P + \gamma\ PV'\right. \tag{7.25}$$

$$H_z^{'} = \frac{-1}{(V-\eta)}\left[H_z\ V' + H_z\ \frac{(\mu-1)}{\mu} + \frac{VH_z\beta}{2\eta}\right] \tag{7.26}$$

$$H_\theta^{'} = -\frac{1}{(V-\eta)}\left[H_\theta\ V' + \left\{\frac{(\mu-1)}{\mu} + \frac{V\beta}{2\eta}\right\}\ H_\theta\right] \tag{7.27}$$

$$V' = \left[\beta\left(P + \frac{(H_\theta + H_z)^2}{2}\right)\frac{V}{\eta} - \frac{(\mu-1)}{\mu}\ (VD\ (V-\eta) - 2P) - (H_\theta + H_z)^2\right]$$

$$-\ P\left(\beta\ \frac{(V-\eta)}{\eta} - \frac{\gamma(2+\beta)V}{2} + \frac{\beta V}{\eta}\right) + (H_\theta + H_z)^2\left(\frac{V}{\eta} - \frac{\beta\ (V-\eta)}{2\eta} + \frac{V-\eta}{\eta}\right)$$

$$\left[D\ (V-\eta)^2 - \gamma\ P + (H_\theta + H_z)^2\right]^{-1} \tag{7.28}$$

Now shock condition 7.14-7.18 are transformed into following form by condition (7.09) and (7.14)

$$u_1 = \frac{2\dot{R}}{(\gamma+1)}$$

$$\dot{R}\ V(\eta) = \frac{2\ \dot{R}}{(\gamma+1)}$$

at $\eta = 1$

$$V(1) = \frac{2}{(\gamma+1)} \tag{7.29}$$

By condition (7.11) & (7.15)

$$P_1 = \frac{2\rho_0 \dot{R}^2}{(\gamma+1)}$$

$$\rho_0 R^2 P(\eta) = \frac{2\rho_0 \dot{R}^2}{(\gamma+1)}$$

at $\eta = 1$

$$P(1) = \frac{2}{(\gamma+1)} \tag{7.30}$$

When the magnetic field is weak J.B. Singh and S.K. Pandey [13].

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)}{(\gamma-1)} \tag{7.31}$$

Which is purely non magnetic By condition (7.10), (7.16) and (7.31)

$$\rho_1 = \frac{(\gamma+1)}{(\gamma-1)} \rho_0$$

$$\rho_0 D(\eta) = \frac{(\gamma+1)}{(\gamma-1)} \rho_0$$

at $\eta = 1$

$$D(1) = 1 \tag{7.32}$$

By condition (7.12) and (7.17), (7.31)

$$h_{\theta_1} = \frac{(\gamma+1)}{(\gamma-1)} h_\theta$$

at $\eta = 1$

$$H_\theta(1) = 1 \tag{7.33}$$

By condition (7.13), (7.18) & (7.31)

$$h_{z_1} = \frac{(\gamma+1)}{(\gamma-1)} h_{z0}$$

$\eta = 1$

$$H_z(1) = 1 \tag{7.34}$$

And appropriate shock condition are

$$V(1) = \frac{2}{(\gamma+1)} \tag{7.29}$$

$$P(1) = \frac{2}{(\gamma+1)} \tag{7.31}$$

$$D(1) = 1 \tag{7.32}$$

$$H_z(1) = 1 \tag{7.33}$$

$$H_\theta(1) = 1 \tag{7.34}$$

**RESULT AND DISCUSSION**

The numerical result for certain choice of parameters are in tabular form. The nature of field variables is in illustrated through tables for following sets of parameter. $\gamma = 4/3,\ 7/5$, $b = 3$, $\mu = 0.7$ $\beta = 0.4$. It is obvious from table that velocity, density, and magnetic fields increase as we move towards the point of explosion while the pressure decrease as we approach towards the centre. The variation in flow variable is negligible for different value of $\gamma$.

**Table- 7.1**

| η | V | D | P | $H_\theta$ |
|---|---|---|---|---|
| 1.000 | 0.757143 | 0.900000 | 0.900000 | 0.900000 |
| 0.999 | 0.762421 | 0.920072 | 0.950051 | 1.033457 |
| 0.998 | 0.767277 | 0.938178 | 0.999549 | 1.066193 |
| 0.997 | 0.771752 | 0.954267 | 0.948722 | 1.098155 |
| 0.996 | 0.775884 | 0.968306 | 0.997786 | 1.129298 |
| 0.995 | 0.779707 | 0.980276 | 0.946948 | 1.159592 |
| 0.994 | 0.783253 | 0.990173 | 0996398 | 1.189015 |
| 0.993 | 0.786547 | 0.098010 | 0.946313 | 1.217557 |
| 0.992 | 0.789617 | 0.903811 | 0.996847 | 1.245220 |
| 0.991 | 0.792284 | 0.907613 | 0.948137 | 1.272016 |

**Table-7.2**

| η | V | D | P | $H_z$ |
|---|---|---|---|---|
| 1.000 | 0.757143 | 1.000000 | 0.900000 | 0.900000 |
| 0.999 | 0.766125 | 1.048126 | 0.920522 | 0.962060 |
| 0.998 | 0.773645 | 1.089802 | 0.941470 | 0.919954 |
| 0.997 | 0.779991 | 1.125024 | 0.964012 | 0.973561 |
| 0.996 | 0.785391 | 1.153950 | 0.989072 | 0.922901 |
| 0.995 | 0.790023 | 1.176868 | 0.917331 | 0.968103 |
| 0.994 | 0.794032 | 1.194148 | 0.949251 | 0.909376 |
| 0.993 | 0.797432 | 1.206212 | 0.985097 | 0.946986 |
| 0.992 | 0.900615 | 1.313508 | 0.424974 | 0.381230 |

**Table-7.3**

| $\eta$ | V | D | P | $H_z$ |
|---|---|---|---|---|
| 1.000 | 0.733333 | 0.900000 | 0.900000 | 0.900000 |
| 0.999 | 0.738495 | 0.916394 | 0.957840 | 0.927960 |
| 0.998 | 0.743311 | 0.931416 | 0.915198 | 0.955445 |
| 0.997 | 0.747810 | 0.945030 | 0.972216 | 0.982419 |
| 0.996 | 0.752018 | 0.957211 | 0.929031 | 0.908850 |
| 0.995 | 0.755959 | 0.967938 | 0.985778 | 0.934714 |
| 0.994 | 0.759653 | 0.977202 | 0.942585 | 0.959989 |
| 0.993 | 0.763123 | 0.984998 | 0.999573 | 0.984661 |
| 0.992 | 0.766386 | 0.991333 | 0.9568/55 | 0.908721 |
| 0.991 | 0.769459 | 0.996217 | 0.914534 | 0.932167 |

**Table-7.4**

| $\eta$ | V | D | P | $H_z$ |
|---|---|---|---|---|
| 1.000 | 0.733333 | 0.900000 | 0.900000 | 0.900000 |
| 0.999 | 0.742203 | 0.940162 | 0.933239 | 0.952128 |
| 0.998 | 0.749851 | 0.975842 | 0.966437 | 0.901436 |
| 0.997 | 0.756477 | 0.906988 | 0.900357 | 0.947808 |
| 0.996 | 0.762249 | 0.933637 | 0.935645 | 0.991201 |
| 0.995 | 0.767304 | 0.955903 | 0.972824 | 0.931637 |
| 0.994 | 0.771756 | 0.973957 | 0.912297 | 0.969188 |
| 0.993 | 0.775699 | 0.988014 | 0.954354 | 0.903970 |
| 0.992 | 0.779212 | 0.998318 | 0.999186 | 0.936128 |
| 0.991 | 0.782362 | 0.905131 | 0.946894 | 0.965828 |

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# Chapter - VIII

# *Similarity Solution of Cylindrical shock wave with radiation energy & material pressure in Magnetohydrodynamics*

## INTRODUCTION

The theory of shock wave and related flows are considerable physical interest Shock waves conceivably driven by solar flares are observed to propagate into interplanetary medium. Rogers [1], Deb Ray [2] have obtained an exact analytic solution for the shock wave problem with an atmosphere of varying density. They have considered the problem taking effect of radiation energy and material pressure shock propagation at very high temperature in which the radiation effects might play a very important role through the coupling of radiation and magnetogasdynamics fields on account of the high temperature, gases are ionised over the entire region of interest in the shock and the medium behaves like a medium of very high electrical conductivity. The explosion along a line in a gas cloud has been discussed

The propagation of shock waves has been studied by Greifigner and Cole [3] and Green span [4], Christer [5] and Ranga Rao & Ramana [6] without taking into account the radiation effects. Elliot [7], wang [8] and Helliwell [9], have considered the effect of thermal radiation in their studies of gas dynamic using similarly method of sedov [10] Singh [11] has studied the problem in ordinary gas dynamics. Theoretical and Experimental studied of radiative shock have been treated by C. Michaut & et al. [12].

In this chapter we consider the problem of cylindrical shock wave in magnetogasdynamics when the atmosphere is non-uniform and conducting taking counter pressure and radiation flux into account. The radiation pressure and radiation energy have been considered. The gas in the undisturbance field is assumed to be at rest and it is grey and opaque.

We suppose that the magnetic field $H_\theta$ and density $\rho_1$ distributions ahead of shock vary as an inverse power of radial distance from centre of symmetry i.e.

$$H_{0_1} = \frac{A}{r}$$

and

$$\rho_1 = \frac{B}{r^w} \left(-2 \leq w \leq 2\right)$$

where A, B and w are constant

## SELF SIMILAR FORMULATION

The cylindrical polar coordinate, where r is the radial distance from axis of symmatry are used here. The equation of conservation of mass, momentum, energy and magnetic flux in the infinite conduction region behind the wave are,

$$\frac{d\rho}{dt} + \frac{\rho}{r}\frac{\partial}{\partial r}(ru) = 0 \tag{8.01}$$

$$\rho\frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\mu H_\theta}{r}\frac{\partial}{\partial r}(rH_\theta) = 0$$

$$\frac{dH_\theta}{dt} + H_\theta\frac{\partial u}{\partial r} = 0 \tag{8.02}$$

$$\frac{dH_\theta}{dt} + H_\theta\frac{\partial u}{\partial r} = 0 \tag{8.03}$$

$$\frac{d}{dt}\left(e+e_r\right) + \left(p+p_r\right)\frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r}\frac{\partial}{\partial r}(rq) = 0 \tag{8.04}$$

$$\rho\frac{\partial u}{\partial t} + \frac{\partial}{\partial r}\left(p+p_r\right) + \frac{\mu H_\theta}{r}\frac{\partial}{\partial r}\left(rH_\theta\right) = 0$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} \tag{8.05}$$

and $\rho$ is the density p the pressure, u the radial velocity, h the azimuthal magnetic field; q the heat flux, t the time and e is the material energy $e_r$ radiation energy $p_r$ radiation pressure. The magnetic permeability $\mu$ is taken to be unity. For an ideal gas we have

$$e = \frac{P\rho}{(\gamma-1)}, \qquad P = \Gamma\rho T \tag{8.06}$$

Where $\gamma$ is the adiabatic gas index, T the temperature and $\gamma$ the gas constant. Assuming local thermodynamic equilibrium and taking Rosseland is diffusion approximation we have

$$q = -\frac{cv}{3}\frac{\partial}{\partial r}\left(\sigma T^4\right) \tag{8.07}$$

where $\frac{\sigma c}{4}$ is stefan Boltzmann constant; C the velocity of light; and v, the mean free path of radiation a function of density and temperature following wang [13], we take

$$v = v_0 \rho^{\alpha} T^{\beta} \tag{8.08}$$

where $v_0$, $\alpha$ and $\beta$ are constant

Given $P = \Gamma \rho T$

By condition (8.07)

$$q = -\frac{cv}{3}\frac{\partial}{\partial r}\left(\sigma T^4\right)$$

Substitute the value T

$$q = -\frac{cv}{3}\frac{\partial}{\partial r}\left(\frac{\sigma P^4}{\Gamma^4 \rho^4}\right)$$

$$v = v_0 \rho^{\alpha} T^{\beta}$$

substitute the value of 'v' in above equation using

$$q = \frac{A^2}{rt} F(\eta)$$

$$\frac{A^2}{rt} F(\eta) = -\frac{c\sigma v_0}{3\Gamma^{4+\beta}}\left\{\frac{A^2 t^2}{r^4} G(\eta)\right\}^{\alpha'-\beta'}\left\{\frac{A^2}{r^2} P(\eta)\right\}^{\beta}\frac{\partial}{\partial r}$$

$$F = N G^{\alpha'-\beta'+4} P^{\beta'+4}\left[\frac{1}{P}\frac{dP}{d\eta} - \frac{1}{G}\frac{dG}{d\eta}\right] \tag{8.09}$$

Where

$$N = \frac{4\sigma c v_0 \rho_c^{\alpha^1-1}}{3\gamma^{(\beta'+4)}} \qquad \alpha^1 = \frac{1}{2} \text{ and } \beta^1 = -3 \tag{8.10}$$

Following Singh & Vishwaka [14] the jump conditions at an isothermal shock front are taken to be

$$\rho_2 = N\rho_1 \tag{8.11}$$

$$P_2 = \frac{N\rho_1 v^2}{\gamma M^2} \tag{8.12}$$

$$H_{\theta_2} = NH_{\theta_1} \tag{8.13}$$

$$u_2 = \left(1 - \frac{1}{N}\right) V \tag{8.14}$$

$$p_{r2} = \frac{N\rho_1 V^2}{\gamma M^2} \tag{8.15}$$

$$q_2 = (N-1)\left[\frac{1}{M_A^2} - \frac{N-1}{2N^2}\right] \rho_1 \ v^3 \ ; \tag{8.16}$$

where

$$N = \left(\alpha + \frac{1}{2}\right) + \left[\left(\alpha + \frac{1}{2}\right)^2 + 2\gamma M^2 a\right]^{1/2}$$

$$\alpha = \frac{1}{\gamma}\left(\frac{M_h^2}{M}\right)$$

In which M and $M_h$ denotes the mach number and Alfven mach number respectively and

$$M^2 = \frac{\rho_1 V^2}{\gamma P_1} \quad and \quad M_h^2 = \frac{\rho_1 V^2}{\mu H_{\theta_1}^2} \tag{8.17}$$

## SIMILARITY SOLUTION

By the standard dimensional analysis of Sedov [10], the non dimensional variables $\eta$ is defined by

$$\eta = \frac{m}{\eta} rt^{-\delta} \tag{8.18}$$

$$\text{where } \delta = \frac{2}{(4-w)}, \quad (w<2) \tag{8.18}$$

$\eta = \left(A^2 / B\right)^{b/2}$ and the dimensionless constant m is defined such that η assumes the value one on the shock front. This choice enables us to write $\eta = \frac{r}{R}$ and $V = \frac{dR}{dt} = \frac{\delta R}{t}$, where R is shock radius.

Then the field variables of the flow variables of the flow pattern in terms of dimensionless function of η are

$$u = \frac{r}{t}\ U(\eta) \tag{8.19}$$

$$P = \frac{A^2}{r^2}\ P(\eta) \tag{8.20}$$

$$H_\theta = \frac{A}{r} H_\theta(\eta) \tag{8.21}$$

$$\rho = \frac{A^2 t^2}{r^4}\ G(\eta) \tag{8.22}$$

$$q = \frac{A^2}{rt}\ F(\eta) \tag{8.23}$$

$$p_r = \frac{A^2}{r^2}\ p_r\ (\eta) \tag{8.24}$$

## SOLUTIONS OF EQUATION OF MOTION

By equation (8.01)

$$\frac{d\rho}{dt} + \frac{\rho}{r}\frac{\partial}{\partial r}(ru) = 0 \tag{8.01}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\rho + \frac{\rho}{r}\frac{\partial}{\partial r}(ru)=0$$

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \frac{\rho}{r}\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \tag{a}$$

By equation (8.19)

$$u = \frac{r}{t} U(\eta)$$

By equation (8.22)

$$\rho = \frac{A^2t^2}{r^4} G(\eta)$$

$$\frac{\partial\rho}{\partial t} = \frac{A^2}{r^4}\left[2tG + t^2 G' \frac{\partial\eta}{\partial t}\right]$$

$$\eta = \left(\frac{m}{n}\right) rt^{-\delta}$$

$$\frac{\partial\eta}{\partial t} = -\delta\left(\frac{m}{n}\right) rt^{-\delta-1}$$

$$\frac{\partial\eta}{\partial t} = -\frac{\delta\eta}{t}$$

then

$$\frac{\partial\rho}{\partial t} = \frac{A^2t}{r^4}\left[2G - G'\,\delta\eta\right]$$

$$\rho = \frac{A^2t^2}{r^4} G(\eta)$$

$$\frac{\partial\rho}{\partial r} = A^2t^2\left[\frac{G'}{r^4}\frac{\partial\eta}{\partial r} - \frac{4}{r^5}G\right]$$

$$\frac{\partial\rho}{\partial r} = \frac{A^2t^2}{r^5}\left[G'\eta - 4G\right]$$

By equation (8.19)

$$u = \frac{r}{t} U(\eta)$$

$$\frac{\partial u}{\partial r} = \frac{1}{t}\left[rU'\frac{\partial\eta}{\partial r} + U.1\right]$$

$$= \frac{1}{t}\left[rU'\frac{\eta}{r} + U\right]$$

$$\frac{\partial u}{\partial r} = \frac{1}{t}\left[U'\eta + U\right]$$

putting these values in equation (a), we get

$$\frac{A^2t}{r^4}\left[2G - G'\,\delta\eta\right] + \frac{r}{t}U\frac{t^2A^2}{r^5}\left[G'\eta - 4G\right] + \frac{A^2t^2}{r^4}G\frac{1}{t}\left[U + U^1\eta\right] + \frac{A^2}{r^4}t^2G\frac{1}{t}U\frac{1}{r} = 0$$

$$2G - G'\,\delta\eta + UG'\eta - 4GU + UG + U'G\eta + GU = 0$$

$$G'\ \eta\,(U-\delta) - 2GU + 2G + U'\,G\eta$$

$$\frac{dG}{d\eta} = \frac{G}{\eta(U-\delta)}\left[2(U-1) - \eta\,\frac{dU}{d\eta}\right] \tag{8.25}$$

Now by equation (8.02)

$$\rho\,\frac{du}{dt} + \frac{\partial P}{\partial r} + \frac{\mu H_\theta}{r}\,\frac{\partial}{\partial r}\,(rH_\theta) = 0$$

$$\rho\,\frac{\partial u}{\partial t} + \rho u\,\frac{\partial u}{\partial r} + \frac{\partial P}{\partial r} + \mu H_\theta\,\frac{\partial H_\theta}{\partial r} + \frac{\mu H_\theta^2}{r} = 0 \tag{b}$$

By condition (8.21)

$$H_\theta = \frac{A}{r}\,H_\theta\,(\eta)$$

$$\frac{\partial H_\theta}{\partial r} = \left[\frac{A}{r}\,H'_\theta\,\frac{\partial \eta}{\partial r} - \frac{AH_\theta}{r^2}\right]$$

$$\frac{\partial H_\theta}{\partial r} = \left[\frac{A}{r}\,H'_\theta\,\frac{\eta}{r} - \frac{A}{r^2}\,H_\theta\right]$$

$$\frac{\partial H_\theta}{\partial r} = \frac{A}{r^2}\,\left[H'_\theta\,\eta - H_\theta\right]$$

By equation (8.20)

$$P = \frac{A^2}{r^2}\,P(\eta)$$

$$\frac{\partial P}{\partial r} = A^2\left[\frac{1}{r^2}\,P'\,\frac{\partial \eta}{\partial r} - \frac{2}{r^3}\,P\right]$$

$$= A^2\left[\frac{P^1}{r^2}\,\frac{\eta}{r} - \frac{2}{r^3}\,P\right]$$

$$\frac{\partial P}{\partial r} = \frac{A^2}{r^3}\left[P'\,\eta - 2P\right]$$

&

$$\frac{\partial u}{\partial r} = \frac{1}{t}\left[U + U'\eta\right]$$

By condition (8.19)

$$u = \frac{r}{t} U(\eta)$$

$$\frac{\partial u}{\partial t} = r\left[\frac{1}{t} U' \frac{\partial \eta}{\partial t} - \frac{1}{t^2} U\right]$$

$$= r\left[\frac{-U'}{t} \frac{\delta \eta}{t} - \frac{U}{t^2}\right]$$

$$\frac{\partial u}{\partial t} = \frac{r}{t^2} \left[-U' \delta \eta - U\right]$$

substituting these values in equation (b)

$$\frac{A^2 t^2}{r^4} G \frac{r}{t^2} \left[-U' \delta\eta - U\right] + \frac{A^2 t^2}{r^4} G. \frac{r}{t} U \frac{(U + U^1 \eta)}{t} + \frac{A^2}{r^3} \left[P' \eta - 2P\right] + \frac{\mu}{r} \frac{A^2}{r^2} H_\theta^2$$

$$+ \mu \frac{AH_\theta}{r} \frac{A}{r^2} \left[H_\theta' \eta - H_\theta\right] = 0$$

$$G \left[-U - U' \delta\eta + U^2 + UU'\eta\right] + \left[P'\eta - 2P\right] + \mu H_\theta H_\theta' \eta = 0$$

$$\frac{dP}{d\eta} = \frac{G}{(U-\delta)} \left[\frac{H^2}{G} - (U-\delta)^2\right] \frac{dU}{d\eta} + \frac{G}{\eta} \left[\frac{2P}{G} - U(U-1)\right] = 0 \tag{8.26}$$

By equation (8.03)

$$\frac{dH_\theta}{dt} + H_\theta \frac{\partial u}{\partial r} = 0 \tag{8.03}$$

$$\frac{\partial H_\theta}{\partial t} + u \frac{\partial H_\theta}{\partial r} + H_\theta \frac{\partial u}{\partial r} = 0$$

By condition (8.21)

$$H_\theta = \frac{A}{r} H_\theta (\eta)$$

$$\frac{\partial H_\theta}{\partial r} = \left[A. \frac{H_\theta'}{r} \frac{\partial \eta}{\partial r} - \frac{1}{r^2} AH_\theta\right]$$

$$\frac{\partial H_\theta}{\partial r} = \frac{A}{r^2} \left[H_\theta' \eta - H_\theta\right]$$

&

$$\frac{\partial H_\theta}{\partial t} = \frac{A}{r} H_\theta^1 \frac{\partial \eta}{\partial t}$$

$$= \frac{A}{r} H_\theta^1 \left( - \frac{\delta\eta}{t} \right)$$

$$\frac{\partial H_\theta}{\partial t} = - \frac{A H_\theta' \delta \eta}{rt}$$

By condition (8.19)

$$u = \frac{r}{t} U(\eta)$$

$$\frac{\partial u}{\partial r} = \left[ \frac{U}{t} + \frac{rU'}{t} \frac{\partial \eta}{\partial r} \right]$$

$$= \left[ \frac{U}{t} + \frac{r}{t} \frac{U'\eta}{r} \right]$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} \left[ U + U'\eta \right]$$

Putting these values in equation (c)

$$-\frac{A H_\theta' \delta\eta}{rt} + \frac{r}{t} U \frac{A}{r^2} \left[ H_\theta' \eta - H_\theta \right] + \frac{H_\theta A}{r} \left[ \frac{U + U'\eta}{t} \right] = 0$$

$$H_\theta' \eta (U - \delta) + U' H_\theta \eta = 0$$

$$\frac{dH_\theta}{d\eta} \eta (u - \delta) + \frac{dU}{d\eta} H_\theta \eta = 0$$

$$\frac{dH_\theta}{d\eta} = \frac{-H_\theta}{(U-\delta)} \frac{dU}{d\eta} \tag{8.27}$$

By equation (8.04) we get

$$\frac{d}{dt} (e + e_r) + (P + P_r) \frac{d}{dt} \left( \frac{1}{\rho} \right) + \frac{1}{\rho r} \frac{\partial}{\partial r} (rq) = 0 \tag{8.04}$$

$$\left( \frac{\partial}{\partial t} + \frac{u\partial}{\partial r} \right) (e + e_r) + (p + p_r) \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \left( \frac{1}{\rho} \right) + \frac{1}{\rho r} r \frac{\partial q}{\partial r} + \frac{q}{\rho r} = 0$$

$$\frac{\partial e}{\partial t} + \frac{\partial e_r}{\partial t} + u \frac{\partial e}{\partial r} + u \frac{\partial e_r}{\partial r} (P + P_r)$$

$$\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial t}+u\,\frac{\partial \rho}{\partial r}\right)+\frac{1}{\rho}\,\frac{\partial q}{\partial r}+\frac{q}{\rho r}=0 \qquad \text{(d)}$$

$$e=\frac{P.\rho}{(\gamma-1)}$$

$$\frac{\partial e}{\partial t}=\frac{1}{(\gamma-1)}\,\frac{\partial}{\partial t}\,(P\rho)$$

$$\frac{\partial e}{\partial t}=\frac{1}{(\gamma-1)}\left[\rho\,\frac{\partial \rho}{\partial t}+\rho\,\frac{\partial \rho}{\partial t}\right] \qquad \text{(i)}$$

&

$$\frac{\partial \rho}{\partial t}=\frac{A^2 t}{r^4}\left[2G-G'\,\delta\eta\right]$$

By condition (8.20)

$$P=\frac{A^2}{r^2}\,P\,(\eta)$$

$$\frac{\partial P}{\partial t}=\frac{A^2}{r^2}\,P'\,\frac{\partial \eta}{\partial t}$$

$$=\frac{A^2}{r^2}P'\left(-\,\frac{\delta\eta}{t}\right)$$

$$\frac{\partial P}{\partial t}=-\,\frac{A^2}{r^2}\,P'\,\frac{\delta\eta}{t}$$

substituting these values in (i)

$$\frac{\partial e}{\partial t}=\frac{1}{(\gamma-1)}\left[\frac{A^2}{r^2}\,P.\,\frac{A^2 t}{r^4}\,\left(2G-G^1\delta\eta\right)-\frac{A^2}{r^4}\,t^2 G\,\frac{A^2}{r^2}\,P'\,\frac{\delta\eta}{t}\right]$$

$$\frac{\partial e}{\partial t}=\frac{A^4 t}{(\gamma-1)r^6}\,\left[2PG-PG'\delta\eta-GP'\delta\eta\right]$$

By equation (8.06)

$$e_r = \frac{(\rho p_r)}{(\gamma - 1)}$$

$$\frac{\partial e_r}{\partial t} = \frac{1}{(\gamma - 1)} \frac{\partial}{\partial t} (p_r \, \rho)$$

$$= \frac{1}{(\gamma - 1)} \left[ P_r \, \frac{\partial \rho}{\partial t} + \rho \, \frac{\partial P_r}{\partial t} \right]$$

By equation (8.24)

$$p_r = \frac{A^2}{r^2} \, P_r \, (\eta)$$

$$\frac{\partial \mathrm{P}_r}{\partial t} = - \, \frac{\mathrm{A}^2}{\mathrm{r}^2} \, \mathrm{P'}_r \, \frac{\delta \eta}{\mathrm{t}}$$

and

$$\frac{\partial \rho}{\partial \mathrm{t}} = \frac{\mathrm{A}^2}{\mathrm{r}^4} \, [2\mathrm{G} - \mathrm{G}^1 \, \delta \eta]$$

By equation (8.22)

$$\rho = \frac{\mathrm{A}^2 \, \mathrm{t}^2}{\mathrm{r}^4} \, \mathrm{G} \, (\eta)$$

so

$$\frac{\partial e_r}{\partial t} = \frac{A^4 t}{(\gamma - 1) r^6} \, [2 P_r \, G - P_r \, G' \, \delta \eta - G \, P_r^1 \, \delta \eta]$$

$$\mathrm{e} = \frac{\mathrm{P}}{(\gamma - 1)} \, \rho$$

$$\frac{\partial \mathrm{e}}{\partial \mathrm{r}} = \frac{1}{(\gamma - 1)} \left[ \rho \, \frac{\partial \mathrm{P}}{\partial \mathrm{r}} + \mathrm{P} \, \frac{\partial \rho}{\partial \mathrm{r}} \right]$$

By equation (8.22)

$$\rho = \frac{A^2 t^2}{r^4} G(\eta)$$

$$\frac{\partial \rho}{\partial r} = A^2 t^2 \left[ \frac{G'}{r^4} \frac{\partial \eta}{\partial r} - \frac{4}{r^5} G \right]$$

$$= A^2 t^2 \left[ \frac{G'}{r^4} \frac{\eta}{r} - \frac{4G}{r^5} \right]$$

$$\frac{\partial \rho}{\partial r} = \frac{A^2 t^2}{r^5} \left[ G' \eta - 4G \right]$$

By equation (8.20)

$$P = \frac{A^2}{r^2} P(\eta)$$

$$\frac{\partial P}{\partial r} = A^2 \left[ \frac{P'}{r^2} \frac{\partial \eta}{\partial r} - \frac{2}{r^3} P \right]$$

$$\frac{\partial P}{\partial r} = \frac{A^2}{r^3} \left[ P' \eta - 2P \right]$$

$$\frac{\partial e}{\partial r} = \frac{1}{(\gamma - 1)} \left[ \frac{A^2}{r^4} t^2 G \frac{A^2}{r^3} (-2P + P'\eta) + \frac{A^2}{r^2} P. \frac{t^2 A^2}{r^5} (G'\eta - 4G) \right]$$

$$\frac{\partial e}{\partial r} = \frac{1}{(\gamma - 1)} \frac{A^4 t^2}{r^7} \left[ -2GP + GP'\eta + G' P \eta - 4GP \right]$$

$$e_r = \frac{p_r \rho}{(\gamma - 1)}$$

$$\frac{\partial e_r}{\partial r} = \frac{1}{(\gamma - 1)} \left[ \frac{\rho \partial}{\partial r} P_r + P_r \frac{\partial \rho}{\partial r} \right]$$

By equation (8.24)

$$P_r = \frac{A^2}{r^2} P_r (\eta)$$

$$\frac{\partial P_r}{\partial r} = \frac{A^2}{r^3} \left[ P_r^1 \eta - 2 P_r \right]$$

and

$$\frac{\partial \rho}{\partial r} = \frac{A^2 t^2}{r^5} \left[G'\eta - 4G\right]$$

$$\frac{\partial e_r}{\partial r} = \frac{1}{(\gamma - 1)} \left[\frac{A^2 t^2 G}{r^4} \frac{A^2}{r^5} \left(-P_r + P_r' \eta\right) + \frac{A^2}{r^2} P_r \frac{t^2 A^2}{r^5} \left(G' \eta - 4G\right)\right]$$

$$\frac{\partial e_r}{\partial r} = \frac{A^4 t^2}{r^7} \left[-GP_r + G P_r' \eta + G' P_r \eta - 4GP_r\right]$$

By equation (8.23)

$$q = \frac{A^2}{rt} F(\eta)$$

$$\frac{\partial q}{\partial r} = \frac{A^2}{t} \left[\frac{F^1}{r} \frac{\partial \eta}{\partial r} + F\left(\frac{-1}{r^2}\right)\right]$$

$$= \frac{A^2}{t} \left[\frac{F'}{r} \frac{\eta}{r} - \frac{F}{r^2}\right]$$

$$\frac{\partial q}{\partial r} = \frac{A^2}{r^2} \left[F'\eta - F\right]$$

substituting these values in equation (d) we get

$$\frac{A^4 t}{(\gamma - 1) r^6} \left[2(P + P_r)G - (P + P_r)G' \delta\eta - G\left(P' + P_r'\right)\delta\eta\right]$$

$$+ \frac{A^4 t^2}{(\gamma - 1) r^7} \frac{r}{t} U. \left[-G(P + P_r) + G\left(P' + P_r'\right)\eta + G'(P + P_r)\eta - 4G(P + P_r)\right] = 0$$

$$\frac{A^2}{r^2} (P + P_r) \frac{r^8}{A^4 t^4 G^2} \left[\frac{A^2 t}{r^4} (2G - G'\delta\eta) + \frac{r}{t} U \frac{t^2 A^2}{r^5} (G'\eta - 4G)\right]$$

$$+ \frac{r^4}{A^2 t^2 G} \frac{A^2}{r^2 t} [F'\eta - F] + \frac{A^2}{rt} F \frac{1r^2}{r. A^2 G} = 0$$

$$\frac{dF}{d\eta} = \frac{G}{(\gamma - 1)} \left\{(U - \delta)^2 - \frac{H_\theta^2}{G} + \frac{\gamma(P + P_r)}{G}\right\}$$

$$-\frac{1}{\eta} \frac{G(U - \delta)}{(\gamma - 1)} \frac{2(P + \mathrm{Pr})}{G} - U(U - 1) + \left(2(P + P_r)U + F\right)$$

Now by equation (8.05)

$$\rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial r}\left(P + P_r\right) + \frac{\mu H_\theta}{r} \frac{\partial}{\partial r}\left(r H_\theta\right) = 0$$

$$\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\partial P_r}{\partial r} + \frac{\mu H_\theta}{r} r \frac{\partial H_\theta}{\partial r} + \frac{\mu H_\theta^2}{r} = 0$$

$$\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\partial Pr}{\partial r} + \mu H_\theta \frac{\partial H_\theta}{\partial r} + \frac{\mu H_\theta^2}{r} = 0 \qquad \text{(e)}$$

By condition (8.19)

$$u = \frac{r}{t} U(\eta)$$

$$\frac{\partial u}{\partial t} = \frac{r}{t^2}\left[-U' \delta\eta - U\right]$$

By condition (8.20)

$$P = \frac{A^2}{r^2} P(\eta)$$

$$\frac{\partial P}{\partial r} = \frac{A^2}{r^3}\left[P'\eta - 2P\right]$$

By condition (8.24)

$$P_r = \frac{A^2}{r^2} P_r(\eta)$$

$$\frac{\partial P_r}{\partial r} = \frac{A^2}{r^3}\left[P_r' \eta - 2 P_r\right]$$

By condition (8.21)

$$H_\theta = \frac{A}{r} H_\theta(\eta)$$

$$\frac{\partial H_\theta}{\partial r} = \frac{A}{r^2}\left[H_\theta' \eta - H_\theta\right]$$

substituting these values in equation (e), we get

$$\frac{A^2 t^2}{r^4} G\left[\frac{r}{r^2}\left(-U' \delta\eta - U\right)\right] + \frac{A^2}{r^3}\left[\left(P' + P_r'\right)\eta\right.$$

$$\left. -2\left(P + P_r\right) + \mu \frac{A}{r} H_\theta \frac{A}{r^2}\left[H_\theta' \eta - H_\theta\right]\right] + \mu \frac{A^2}{r^2} H_\theta \frac{2}{r} = 0$$

$$G\left(-U'\delta\eta-U\right)+\left(P'+P_r'\right)\eta-2\left(P+\mathrm{Pr}\right)$$

$$+\mu H_\theta\left(H_\theta^1\eta-H_\theta\right)+\mu H_\theta^2=0$$

Putting of values $\mu=1$ and $H'_\theta$

$$\frac{du}{d\eta}=\frac{2(P+P_r)}{G}(U-2\delta+1)-U(U-1)(U-\delta)+\frac{F}{K}\frac{G^{\beta-\alpha+3}}{\left(P^{\beta+3}+P^{1\beta+3}\right)}(U-\delta) \tag{8.29}$$

$$\left[(U-\delta)^2-\frac{H_\theta^2}{G}+\frac{(P+P_r)}{G}\right]$$

where $k=\dfrac{4c\sigma v_0}{3\Gamma A}$

a non dimensional radiation parameter.

The transformed shock conditions are, using equation (8.11-8.16) & (8.19-8.24) & (i), (ii) By condition (8.14) & (8.19)

$$U_2=\left(1-\frac{1}{N}\right)V$$

$$\frac{r}{t}U(\eta)=\left(1-\frac{1}{N}\right)\frac{\delta R}{t}$$

$r=R$ at $\eta=1$

$$\frac{R}{t}U(1)=\left(1-\frac{1}{N}\right)\frac{\delta R}{t}$$

$$U(1)=\left(1-\frac{1}{N}\right)\delta \tag{8.30}$$

By condition (i), (8.13) & (8.21)

$$H_{\theta_2}=NH_{\theta_1}$$

$$\frac{A}{r}H_\theta(\eta)=N.\frac{A}{r}$$

at $\eta=1$

$$H_\theta(1)=N \tag{8.31}$$

By condition (8.11) & (8.22)

$$\rho_2=N\rho_1$$

$$\frac{A^2t^2}{r^4}\,G(\eta) = N\,\rho_1$$

$$\frac{A^2t^2}{r^4}\,G\,(\eta) = N.\frac{Mh^2}{v^2}\,H_\theta^2,\,\mu$$

$$\frac{A^2}{r^4}\,t^2\,G\,(\eta) = N.M_h^2\,\frac{A^2}{r^2}\,\frac{t^2}{\delta^2 R^2}$$

$R = r,\ \eta = 1$

$$G\,(1) = \frac{N M_h^2}{\delta^2} \qquad (8.32)$$

By condition (8.12) & (8.20)

$$P_2 = \frac{N\rho_1\,v^2}{\gamma M^2}$$

$$\frac{A^2}{r^2}\,P\,(\eta) = \frac{N.M_h^2}{\gamma\,M^2}\,\mu\,H_\theta^2$$

$$= \frac{N.\,M_h^2}{\gamma M^2}\,\mu\,\frac{A^2}{r^2}$$

$\mu = 1$ & $\eta = 1$

$$P\,(1) = \frac{N.\,M_h^2}{\gamma M^2} \qquad (8.33)$$

By condition (8.15) & (8.24)

$$P_{r_2} = \frac{N\rho_1\,v^2}{\gamma m^2}$$

$$\frac{A^2}{r^2}\,P_r\,(\eta) = \frac{N.\,M\,\mu H_\theta^2}{\gamma M^2}$$

$\eta=1,$ & $\mu=1$

$$p_r\,(1) = \frac{NM_h^2}{M^2}$$

By condition (8.16) & (8.23)

$$q_2 = (N-1)\left[\frac{1}{M_h^2} - \frac{N-1}{2N^2}\right] \rho_1 v^3$$

$$\frac{A^2}{r.t} F(\eta) = (N-1)\left[\frac{1}{M_h^2} - \frac{N-1}{2N^2}\right] M_h^2 \, \mu H_\theta . v$$

$at\, \eta = 1, \;\&\; \mu = 1$

$$F(1) = L M_h^2 \, \delta \tag{8.35}$$

where

$$L \;\; (N-1)\left[\frac{1}{M_h^2} - \frac{N-1}{2N^2}\right]$$

as the initial values for our numerical calculation

## RESULTS AND DISCUSSION

In order to exhibit the numerical solution it is convenient to write the field variables in the non dimensional form as

$$\frac{u}{u_2} = \eta \frac{U(\eta)}{\left(1 - \frac{1}{N}\right)\delta} \tag{8.36}$$

$$\frac{\rho}{\rho_2} = \frac{\delta^2}{\eta^4 \, N M_h^2} \; G(\eta) \tag{8.37}$$

$$\frac{P}{P_2} = \frac{1}{\eta^2} \frac{\gamma M^2}{N M_h^2} P(\eta) \tag{8.38}$$

$$\frac{Pr}{Pr_2} = \frac{1}{\eta^2} \frac{\gamma M^2}{N M_h^2} Pr(\eta) \tag{8.39}$$

$$\frac{H_\theta}{H_{\theta_2}} = \frac{1}{\eta} \frac{1}{N} H(\eta)$$

$$\frac{q}{q_2} = \frac{1}{\eta \, M_h^2 \, \delta} \eta^2 \frac{P(\eta)}{G(\eta)} \tag{8.40}$$

$$\frac{T}{T_2} = \gamma \left(\frac{M}{\delta}\right)^2 \eta^2 \frac{P(\eta)}{G(\eta)} \tag{8.41}$$

By using equation. We have calculated our result for the following set of parameters.

$\gamma = 1.4$ $M_h^2 = 20$, $M^2 = 10$ $\delta = 5, 2$ and $k = 10$ (8.42)

The numerical results for certain choice of parameter are reproduced in tabular form. The nature of field variables is illustrated through table 8.01 and 8.02.

We can easily see that the radiation parameters has an important effect on the flow variables magnetoradiative effects are prominent on the field variables when we compare our results with the results of ordinary gas dynamics.

**Table-8.01**

$K = 10, \delta = 2$

| $\eta$ | **u/u$_2$** | $\rho/\rho_2$ | **p/p$_2$** | **H/H$_2$** | **q/q$_2$** | **T/T$_2$** |
|---|---|---|---|---|---|---|
| 1.00 | 1.200000 | 1.200000 | 1.200000 | 1.200000 | 1.200000 | 1.200000 |
| 0.99 | 1.189713 | 1.211770 | 1.233358 | 1.259207 | 2.205242 | 1.201495 |
| 0.98 | 1.179343 | 1.237695 | 1.252275 | 1.288322 | 3.225350 | 1.104061 |
| 0.97 | 1.168883 | 1.258038 | 1.277232 | 1.317315 | 4.252065 | 1.207675 |
| 0.96 | 1.158326 | 1.283072 | 1.298743 | 1.346149 | 6.297158 | 1.212310 |
| 0.95 | 1.147663 | 1.303076 | 1.317355 | 1.374785 | 7.312421 | 1.217943 |
| 0.94 | 1.136886 | 1.328336 | 1.343649 | 1.413179 | 8.339657 | 1.224552 |
| 0.93 | 1.125985 | 1.349134 | 1.388237 | 1.431281 | 10.350662 | 1.232112 |
| 0.92 | 1.114949 | 1.355754 | 1.391761 | 1.459036 | 11.377211 | 1.240601 |
| 0.91 | 1.103765 | 1.378466 | 1.424887 | 1.496383 | 13.491037 | 1.249999 |
| 0.90 | 1.192420 | 1.397525 | 1.454297 | 1.523254 | 15.513795 | 1.260283 |
| 0.89 | 1.180898 | 1.423162 | 2.512686 | 1.549573 | 17.537041 | 1.271435 |
| 0.88 | 1.169183 | 1.435570 | 2.538742 | 1.575256 | 18.55182 | 1.283434 |
| 0.87 | 1.157254 | 1.464890 | 2.557137 | 1.590211 | 20.570432 | 1.296262 |
| 0.86 | 1.145091 | 1.491199 | 2.578501 | 1.614334 | 22.612750 | 1.209902 |

**Table-8.02**

K = 10, δ = 5

| η | u/u2 | ρ/ρ2 | p/p2 | H/H2 | q/q2 | T/T2 |
|---|---|---|---|---|---|---|
| 1.00 | 1.200000 | 1.200000 | 1.200000 | 1.200000 | 1.200000 | 1.200000 |
| 0.99 | 1.180721 | 1.227090 | 1.308584 | 1.402352 | 1.557591 | 1.551374 |
| 0.98 | 1.171376 | 1.252716 | 1.356953 | 1.424803 | 2.575334 | 1.593582 |
| 0.97 | 1.161957 | 1.277757 | 1.456264 | 1.457320 | 4.597607 | 1.616606 |
| 0.96 | 1.152454 | 1.303175 | 1.487805 | 1.479862 | 5.619113 | 1.650427 |
| 0.95 | 1.132855 | 1.350017 | 1.493009 | 1.502382 | 6.664906 | 1.705027 |
| 0.94 | 1.093147 | 1.459420 | 1.534625 | 1.534821 | 7.690399 | 1.720391 |
| 0.93 | 1.083317 | 1.492608 | 1.550913 | 1.637111 | 9.701375 | 1.766503 |
| 0.92 | 1.073348 | 1.530890 | 2.597283 | 1.699173 | 10.733997 | 1.793348 |
| 0.91 | 1.053221 | 2.585648 | 2.614659 | 1.710912 | 12.754799 | 1.820913 |
| 0.90 | 1.042917 | 2.628327 | 2.655302 | 1.752220 | 13.790675 | 1.859184 |
| 0.89 | 1.032411 | 2.666048 | 2.701621 | 1.782970 | 15.808854 | 1.878150 |
| 0.88 | 1.021676 | 2.693373 | 3.736155 | 1.793013 | 17.826841 | 1.897801 |
| 0.87 | 1.010680 | 3.718656 | 3.751529 | 1.822180 | 19.852347 | 1.908128 |
| 0.86 | 1.009388 | 3.737572 | 3.790382 | 1.880276 | 21.893165 | 1.929125 |

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## PRESENTED IN CONFERENCES

1. Vijnana Parishad of India 12th "Annual Conference and National Symposium on Applications of Special Functions". Kishore Kumar Srivastava and Jitendra Kumar Held at department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur, Rajasthan on October 25-27, 2007.

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# PUBLICATIONS

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2. Topic "Analysis of self similar motion in the theory of stellar explosion" is proceeding in **"Ganit Sandesh", Rajasthan Ganita Parishad, Rajasthan** (Under Process).
3. Topic "Similarity solution of isothermal shock with radiative heat flux" in **Acta Cinecia Indica, Meerut** (Accepted).
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